THE IMPACT OF INDIVIDUAL CURRICULA ON TEACHING STOCHASTICS

Andreas Eichler, Institut für Didaktik der Mathematik, TU Braunschweig

This report focuses on teachers' individual curricula. An individual curriculum includes contents and reasoning and can be structured in a quasi-logical system of goals and methods, which is the result of teachers' planning of mathematics instruction. There is consent that the planning of individual curricula or the instructional practice is a form of social action. While action is an inner and subjective process, which is dependent on situations and individuals' interpretation of a situation, here, the approach of research is qualitative and interpretative. So, individual curricula are re-constructed from interviews held with eight secondary teachers. One of the eight cases, the one of Mr. A, is lined out here.

THEORETICAL FRAMEWORK

Curricula change. Here, curriculum means the system of subjects of instruction and reasoning of this system and issues that are directly related. While the contents of teaching as well as the main goals of teaching different mathematical disciplines are similar or identical, the importance of specific contents and the system of reasoning depend on the development of theories of teaching (mathematics). As the development of new curricula follows social or political requirements or new didactical knowledge, it must be the main goal of professionals responsible for developing curricula to realize new ways of learning and teaching in schools. In every learning theory, the key persons to apply new curricula to enable students to acquire (subjective) knowledge are teachers (see Fernandes 1995 and Wilson/Cooney 2002). Only they choose the subjects of instruction and only they define their goals for teaching mathematics.

There are – especially in Germany – two ways of realizing new curricula. In the revolutionary way, curricula are published by state governments and have to be installed in schools. But, especially in Germany, research shows that this way does not work. Governmental curricula and even didactical proposals for modified curricula are obviously realized seldom in the daily instruction of mathematics (see Vollstädt et al. 1998). This is (in Germany) especially the case for stochastics. Over 40 years, it is a didactical demand to teach stochastics or to teach more stochastics, but stochastics are hardly present in today's mathematics instruction. So what may be the key of changing mathematics and stochastics instructional practice?

In the evolutionary way, teachers integrate step by step didactical proposes in their individual curricula in an active and self-determined way. Here, it is one main hypothesis that understanding teachers' individual curricula is mandatory to grasp their instructional practice and to be able to change this practice (see also Pehkonen/Törner 1993). So teachers' individual curriculum, their subjective knowledge and concep-

Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, 2004 Vol 2 pp 319–326

tions about mathematics and about learning and teaching mathematics is the focus of this line of research.

Governmental and didactical curricula consist of contents and their reasoning. Reasoning means that methods and goals are connected in form of if-then-sentences (see König 1975). One example:

Students have to learn data-analysis. (goal)		
If students learn data-analysis	*then*	they will become an individual with the ability to criticize. (method)
		Students have to become an individual with the ability to criticize. (goal)

Figure 1: Goals and methods

Here, it is another main hypothesis that individual curricula are constructed in the same way. The system of reasoning called goal-method-argumentation shall broaden and deepen the results of research on beliefs postulating types of teachers and their individual curricula (see Thompson 1984, Thompson 1992 and Leder, Pehkonen and Törner 2002).

There is consent that the planning of individual curricula or the instructional practice is a form of social action. While action – in sociological as in psychological definition – is an inner and subjective process, which is dependent on situations and individuals' interpretation of a situation (see Wilson 1973), here, the approach of research is qualitative and interpretative and uses the well elaborated psychological approach of subjective theories (see "Forschungsprogramm Subjektive Theorien", Groeben et al. 1988). This approach newly developed in contrast to the behavioristic way by understanding people's acting. It proposes the epistemological modelling of human theories of actions, which are parallel to researchers' theories. The approach is oriented on Kelly's (1955) report about the "man as scientist" and focuses on people's subjective knowledge structured in quasi-logical systems of concepts. Groeben et al. (1988) postulate the goal-method-argumentation to be one of these quasi-logical systems of concepts.

So, the question of research focused on in this report is:

What are the contents and goals of teachers' individual curricula of stochastics?

What are the goal-method-argumentations of teachers' individual curricula and how they structure teachers' instructional goals?

METHODOLOGY

The approach of understanding action as an inner process depending on situations determines an inquiry in form of case studies (see Stage 2000). The definition of the cases is according to the theoretical sampling (see Glaser/Strauss 1967). Here, the cases were eight teachers grade 7 to 13 of secondary schools (A-level)[1] in Northern Germany. The reconstruction of the individual (stochastic) curricula is based on interviews designed in form of the problem-oriented interview (see Witzel 1982). The topics of these interviews emerged form the analysis of didactical curricula.

The interviews were taped and transcribed. The interpretation was done according to the main method of humanities, the hermeneutics (see Gadamer, H. G. 1986 and Danner, H. 1998). Firstly, subjective concepts or goals of instruction and their definitions were reconstructed. The second step of reconstruction included the reduction of subjective concepts to main concepts and the construction of a system of goals and methods and their relations in form of if-then-sentences (goal-method-argumentation). This step determined the differentiation between five aspects of an individual curriculum: The contents of instruction, the goals of stochastics and mathematics instruction, teachers' knowledge about how students view the usefulness of mathematics and finally, teachers' knowledge about teaching mathematics successfully (see to the latter two also Brown 1995). The identification of patterns of argumentations and the definitions of types across individual curricula is not subject of this report.

The following discussion focuses on the results and their interpretation. The process of interpretation of the interviews and the reconstruction is not lined out. While primarily one case will be discussed, some results of the other cases are used to complete the case description.

THE CASE OF MR. A

A is a teacher at a gymnasium in a little town in Northern Germany. As only three of twelve of A's colleagues teach stochastics, A has to start with elementary fundamentals of stochastics in grade 10 and grade 13, where he teaches stochastics.

The curriculum concerning the subjects of instruction is shown below (see figure 1).

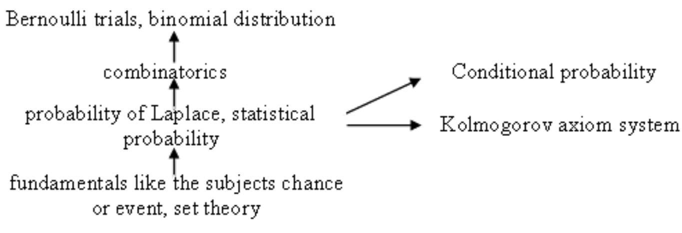

Figure 2: Subjects of instruction

There are four characteristic aspects to this curriculum. Firstly, A defines school-stochastics as theory of probability. Elements of data-analysis such as descriptive statistics or statistical inference are missing. Furthermore, the curriculum is limited. Up from elementary fundamentals it will end with, also in the highest grade, an introduction to binomial distributions. Some of these contents (see figure 1) are part of all individual curricula analysed and are summed up to the category of a classic block of theory of probability. This means the sequence: fundamentals (for example chance or event), probability, combinatorics, Bernoulli-experiment and binomial distributions.

The interpretation of the central idea of probability (the statistical or subjective interpretation or the interpretation of probability defined by Laplace or described by the axioms) has evolved to be a main criterion of the curriculum's analysis. There is one type of teacher like A, who anchors her or his curriculum in Laplace's interpretation of probability and limits the curriculum as shown above. For example, another type of teacher focuses on the statistical interpretation of probability. Her or his individual curriculum includes data-analysis and especially statistical inference (the differentiation is independent of the time-span teachers use for teaching stochastics). Instead of teaching statistics, A also covers subjects like Kolmogorov's system of axiom or the conditional probability, which are not necessary for the main curriculum. Especially teaching axioms fulfils A's idea of gymnasium's instruction of mathematics. So, on the one side A's curriculum is limited, but on the other side it is extensive within its limits.

These characteristics are integrated in the argumentation on the goals of teaching stochastics (figure 2 and also the following figures show a strongly condensed version of the goal-method-argumentations).

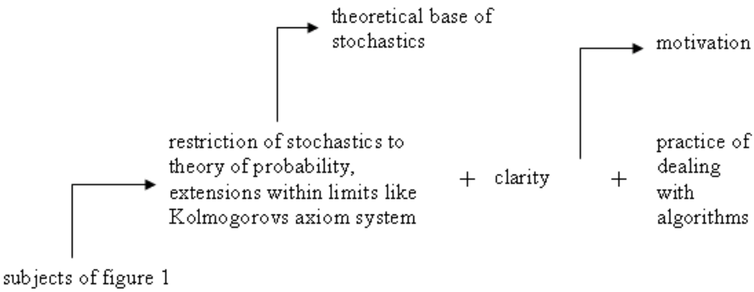

Figure 3: Goals of stochastics curriculum

The link between contents and goals of instruction is in every case one goal based on the thesis that the contents of instruction are the result of the teacher's conscious election. In the first level of goals there are some, which are concerning contents of instruction and are described above. Other goals include reasoning of instructional practice like using clear concepts in instruction, which are not based on special subjects of instruction and are meant to increase motivation. Here, motivation means students enjoyed doing mathematics and so efficient instruction is possible. Finally, there is the goal, especially for weak students, to acquire the ability to use mathematical algorithms.

The subject-oriented goals are the main goals in A’s goal-method-argumentation. So the main goal of stochastics is to build up a theoretical base of stochastics. This base does not enable students to solve real stochastic problems. It is only a base, which may be extended after leaving school. Beside stochastics algorithms it is the formal system of stochastics, axioms, definitions, theorems and proofs, which characterizes the theoretical base.

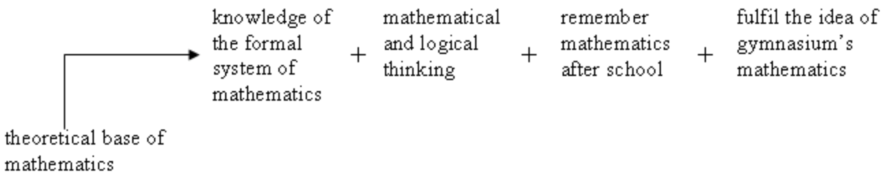

Figure 4: Goals of mathematics curriculum

As the theoretical base is the core of A’s individual stochastics curriculum, it is the core of the mathematics curriculum, too. The knowledge of the formal and deductive system of mathematics according to school-mathematics and the ability of dealing with special mathematical algorithms is the prerequisite to achieve the highest goals of school-mathematics:

the knowledge of the formal and deductive system of mathematics;

the ability of mathematical and logical thinking. This means the ability to make deductive conclusions and to think of assumptions and conclusions;

the latter goals should be achieved in gymnasium’s mathematics;

at last, it is A’s opinion that only a solid theoretical base enables students to remember the mathematical subjects, methods, relations after their school-career.

For A, school-mathematics in general does not enable students to deal with real problems. While A’s main goal is to convey the formal system of mathematics other teachers define goals concerning problem-solving or dealing with real problems.

In the context of another goal-method-argumentation, A’s subjective knowledge about how students view the usefulness of mathematics is anchored in a goal discussed above, the ability of dealing with mathematical algorithms. This argumentation (see figure 4) is a pragmatic one. So students need this ability to manage school examinations. If they are successful, they obtain the permission to attend a university. If students view themselves as prepared for life they are satisfied with school, which is the highest goal in this argumentation.

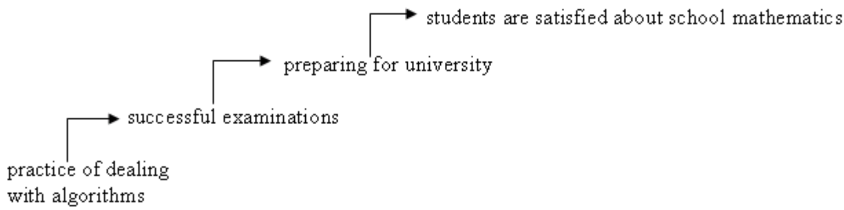

Figure 5: Students view on the usefulness of school-mathematics

In other cases, this argumentation does not only apply to prepare students for university or profession but also to prepare for life in terms of the ability of criticism.

A's last goal-method-argumentation (see figure 5) concerns his subjective knowledge about the efficiency of instructional practice. As sufficient exercise in dealing with mathematical algorithms results in success for most students, the clarity of instruction and, at last, an atmosphere of respect and understanding, however, makes the learning and teaching of mathematics possible. Firstly, it leads students to motivation. A's definition of three classes of efficient instructional practice is striking. Firstly, efficient instructional practice only means that students work and learn mathematical content. If this content has real applications or opens a deep insight into the formal system of mathematics, then A defined this as meaningful instruction. The third class is the combination of the latter two classes. A termed this worth-while instruction and stated that it is seldom realized.

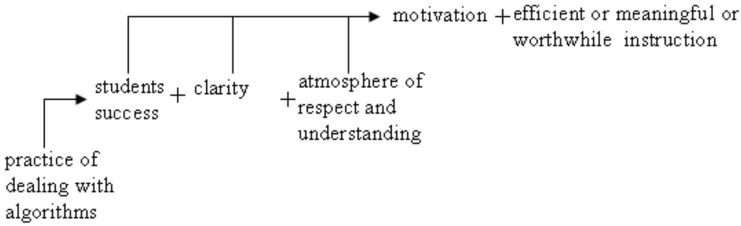

Figure 6: Efficiency of the instructional practice

DISCUSSION

A's individual curriculum – and also the other individual curricula – with special regards to stochastics consists of the discussed five aspects. The base of all goal-method-argumentations is the system of the contents of instruction. It is impossible to understand one's goals of stochastics or mathematics without knowing this base. The other two argumentations concerning content-oriented goals of stochastics and mathematics are well matched in the case of A. The other argumentations concerning students' use of mathematics and the functionality of teachers' instructional practice are separate from special mathematical subjects. Finally, the following theses are based on this case description:

The five aspects of an individual curriculum discussed here are the main aspects of every individual curriculum.

In the eight individual curricula analysed there were no subjective concepts (or goals) that could not be ascribed to one of the five aspects. Also these five aspects are similar to theoretical differentiations of aspects of school-mathematics (see for example Thompson 1992).

Only the knowledge of all aspects leads to a real understanding of an individual curriculum.

One example: There is an open conflict in A's curriculum concerning the teaching of the formal system of mathematics or the algorithms as a toolbox. The main goal of stochastics and mathematics seems to be teaching the formal and deductive system. But the other argumentations show that for students' success and for efficient instruction it is necessary to reduce formalism and extend the exercise of algorithms. Furthermore, without knowing his goals oriented at the formal system it is impossible to understand some of A's contents of instruction like his teaching of the axioms or the conditional probability. So only the comprehensive analysis opens a real and deep understanding of A's individual curriculum.

Without teachers it is impossible to implement new ideas or developments of didactical curricula into schools instructional practice.

This hypothesis based on theoretical considerations and empirical results lined out in the discussion of the theoretical framework.

Without understanding teachers' individual curricula it is impossible to change these curricula.

One example: One new development of didactical curricula of stochastics is the extension to data-analysis. This could prove to be difficult for A, since there seems to be no anchor for integrating data-analysis into A's individual curriculum.

While teachers have to consider students' individual knowledge, also didactical curricula have to consider teachers' subjective knowledge and their individual curricula.

References:

Brown, L.(1995), The Influence of Teachers on Children's Image of Mathematics. Proceedings of the 19th PME Conference, Vol. 2, 146-153.

Cooney, T.J., Shealy, B.E., Arvold, B. (1998), Conzeptualizing Belief Structures of Preservice Secondary Mathematics Teachers. In Journal for Research in Mathematics Education 3, 306-333.

Cooney, T. J., Wilson, M. (2002), Mathematics teacher change and development. In Leder, G.C., Pehkonen, E., Törner, G. (ed.) (2002), Beliefs: a hidden variable in mathematics education? Dodrecht: Kluwer, 127-147.

Danner, H. (1998), Methoden geisteswissenschaftlicher Pädagogik. Tübingen: Mohr.

Fernandes, D. (1995), Analysing four preservice teachers' knowledge and thoughts through their biographical histories. Proceedings of the 19th PME Conference, Vol. 2, 162-169.

Gadamer, H. G. (1986): Wahrheit und Methode. Tübingen: Mohr.

Glaser, B., Strauss, A. (1967), The discovery of grounded theory. Chicago: Aldine.

Groeben, N., Scheele, B., Schlee, J., Wahl, D. (1988), Das Forschungsprogramm Subjektive Theorien. Tübingen: Francke.

Kelly, G.A. (1955), The psychology of personal constructs. New York: Norton.

König, E. (1975), Theorie der Erzierhungswissenschaft, Vol. 1-3. München: Wilhelm Fink 1975.

Leder, G.C., Pehkonen, E., Törner, G. (ed.) (2002), Beliefs: a hidden variable in mathematics education? Dodrecht: Kluwer.

Pehkonen, E., Törner, G. (1993). On teachers' beliefs and changing mathematics teaching. In Journal für Mathematikdidaktik 3/4, 177-209.

Stage, R. E. (2000), Case studies. Denzin, N. K., Lincoln, Y. S. (ed.), Handbook of Qualitative Research, Thousend Oaks, London, New Dehli: Sage Publications, 435-508.

Thompson, A. G. (1984), The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. In Educational studies in mathematics 15, 105-127.

Thompson, A. G. (1992), Teachers' beliefs and conceptions: a synthesis of the research. In Grouws, D. A. (ed.) Handbook of research on mathematics teaching and learning. New York: MacMillan.

Vollstädt, W., Tillmann, K. J., Rauin, U., Höhmann, K., Tebrügge, A. (1998), Lehrpläne im Schulalltag. Opladen: Leske + Budrich.

Wilson, T. P. (1973), Theorien der Interaktion und Modelle soziologischer Erklärung. In Arbeitsgruppe Bielefelder Soziologen (ed.), Alltagswissen, Interaktion und gesellschaftliche Wirklichkeit. Hamburg: Reinbek, 54-79.

Witzel, A. (1982), Verfahren der qualitativen Solzialforschung, Frankfurt a.M., New York: Campus.

---

[1] A-level means the German gymnasium. In Germany, there are generally three sorts of schools, the Hauptschule, the Realschule and the Gymnasium. The Gymnasium is a school with students from grade 7 to grade 13. If students manage the second level (grade 11 to grade 13), they are entitled to attend a university.