DESCRIBING ELEMENTS OF MATHEMATICS LESSONS THAT ACCOMMODATE DIVERSITY IN STUDENT BACKGROUND

Peter Sullivan — La Trobe University
Judy Mousley — Deakin University
Robyn Zevenbergen — Griffith University

We are researching actions that teachers can take to improve mathematics learning for all students. Structural elements of the lessons being trialled include making aspects of pedagogy explicit to seek to overcome differences in familiarity with schooling processes, and sequencing tasks with the potential to engage students. This article reports research on teachers building learning communities by preparing variations to set tasks in order to address differences in students' backgrounds.

THREE KEY LESSON ELEMENTS

All lessons are taught to students at different stages of readiness to learn. These differences can be a result of variations in motivation (Middleton, 1995), persistence (Dweck, 2000), perceptions of the value of schooling (Deplit, 1988), social group or cultural factors, or varying degrees of necessary experiences in particular mathematical domains. What is not clear is how teachers can structure lessons to respond to these differences. We agree with Boaler (2003) who argued that:

> researchers in mathematics education need to study the practice of classrooms in order to understand relationships between teaching and learning and they need to capture the practices of classrooms in order to cross divides between research and practice. (Boaler, 2003, p. 15)

The following discussion uses an illustrative lesson, structured in a particular way, as data. The general lesson structure is an outcome of our ongoing research into characteristics of lessons that are successful with students from diverse backgrounds. The 3 key elements that contribute to this lesson structure are:
- *explicit pedagogies*, or teacher actions to be articulated or enacted explicitly;
- *a set of mathematical tasks*, with an open-ended "goal task", sequenced to ensure that all students have the necessary experience to be successful at each stage;
- *the building of a learning community*, evident in the way student products are reviewed and variations offered to students based on their responses to the tasks.

The first two elements are based on the socio-mathematical framework of Cobb and others (e.g., Cobb & McClain, 1999), where two complementary norms of activity are delineated. *Socio-cultural norms* are the practices, organisational routines, and modes of communication that impact on approaches to learning, types of responses valued, views about legitimacy of knowledge produced, and responsibilities of individual learners. *Mathematical norms* are principles, generalisations, processes, and products that form mathematics curriculum and serve as learning tools. This article focuses on the third element, which is a focus of our research. It refers to ways of catering for differences in background experiences of students. This paper reports on one approach to this challenge: varying the tasks posed to support the participation of all students in order to build a "learning community".

Explicit pedagogies

Various authors have commented on aspects of schooling that tend to exacerbate the obvious difficulties that some students experience. For instance, Anyon (1981) focused on the ways mathematics learning tasks are posed; Mellin-Olsen (1981) proposed that features of the social context influence learning goals and strategies adopted by pupils; Lerman (1998) attended to SES-related differences between classroom expectations and students' aspirations; and Zevenbergen and Lerman (2001) argued that the ability to decode contextualised problems corresponds closely with students' SES backgrounds. Our investigations have identified further areas that are likely to create difficulties for some students, including the use of particular language genres; the use of contexts that proved inappropriate for some groups of students; and lack of explanation of the purposes of different teaching strategies, or contexts, or expectations for student engagement in various types of tasks (Sullivan, Mousley, Zevenbergen, & Turner-Harrison, 2003). We see these as directly connected to the socio-cultural norms, and have shown how it is possible for teachers to address these norms by being more explicit about various aspects of pedagogical practice.

A Set of Mathematical Tasks

It is proposed that student engagement comes from working on sequenced problem-like tasks, rather than following teachers' step-by-step instructions. There are two reasons for this. The first is recognition that learning and knowing are products of activity that is "individual and personal, and … based on previously constructed knowledge" (Ernest, 1994, p. 2). The second relates to the role of the teacher in identifying blockages, prompts, supports, challenges and pathways. Cobb and McClain (1999) argued that teachers should form an "instructional sequence …a conjectured learning trajectory that culminates with the mathematical ideas that constitute our overall instructional intent" (p. 24).

While not essential to the lesson structure being researched, we use open-ended tasks where possible, since they are likely to foster the type of engagement sought while allowing the teacher to plan for student learning. Open-ended tasks are generally more accessible than closed examples, as students who experience difficulty with traditional questions can approach tasks in their own ways (see Sullivan, 1999).

The Building of a Learning Community

Many lessons seem to either ignore the diversity of students' backgrounds and needs, or address them in ways that exacerbate difference by having alternative goals for particular groups of students. Our intention is that all students engage sufficiently in a lesson to allow them to participate fully in a whole-class review of their work on the goal task. Students may follow different pathways to the ultimate task, being supported or detoured along the way, but all will feel part of the classroom community. All will know they are expected to master the content, and it is expected that their engagement in discussions will offer substantial educative opportunities.

Sometimes school communities seek to address differences in student achievement by grouping students of like achievement together. It seems that the consensus is that this practice has the effect of reducing opportunities especially for students placed in the lower groups (Boaler, 1997; Zevenbergen, 2003). This can be partly due to self-fulfilling prophesy effects (e.g., Brophy, 1983), and partly due to the effect of teacher self-efficacy which is the extent to which teachers believe they have the capacity to influence student performance (e.g., Tschannen-Moran, Hoy, & Hoy, 1998). Brophy argued that, rather than grouping students by their achievement, teachers should: concentrate on teaching the content to whole class groups rather than worrying too much about individual differences; keep expectations for individuals current by monitoring progress carefully; let progress rates rather than limits adopted in advance determine how far the class can go; prepare to give additional assistance where it is necessary; and challenge and stimulate students rather than protecting them from failure or embarrassment.

One key aspect of this approach relates to the supports offered to students who experience difficulty along the way. It is common, indeed in places recommended, that teachers gather such students together and teach them at an appropriate level (see, for example, Department of Education, Employment and Training, 2001). We suggest that a sense of community is more likely to result from teachers offering prompts to allow students experiencing difficulty to engage in active experiences related to the initial goal task, rather than, for example, requiring such students to listen to additional explanations, or to assume that they will pursue goals substantially different from the rest of the class. Adapting carefully selected core tasks to provide appropriate problem solving opportunities to students experiencing difficulty is evident in the recommendations of Griffin and Case (1997), Ginsburg (1997), and Thornton, Langrall and Jones (1997).

Related to this is ensuring that students who finish tasks quickly, at any stage along the way, are posed supplementary tasks that extend their thinking on that task, rather than proceeding onto the next stage in the lesson. One characteristic of open-ended tasks is that they create opportunities for extension of mathematical thinking, since students can explore a range of options as well as consider forms of generalised response. Unless creative opportunities are provided for the students who have completed the tasks along the way then not only can they be bored, and so create difficulties for the teacher, but also they will not be using their time effectively.

Thus the premise of our notion of learning community is that the class progresses through to a common goal task of the lesson, developing a set of understandings than can create the context for whole-class discussion, synthesis, and lesson closure, then form a shared basis for following lessons.

THE LESSON DESCRIPTION: AREA AS COUNTING SQUARES

Lesson descriptions constitute our raw data. The lesson referred to below is an example of lessons that were prepared to incorporate each of the desired planning elements. It was taught, with observers present, on three occasions to upper primary grades (ages 10 to 12): Class A was a small group of Indigenous Australian students; students in Class B were from a predominantly lower SES community in a provincial Australian city; and students in Class C were from a large regional centre with mixed SES backgrounds. In each lesson, an observer completed an instrument adapted from that used by Clarke et al. (2002) which consisted of (a) a naturalistic report using two columns, one for a record of what happened and one for the observers' impressions; (b) a form for overview comments; and (c) a framework to structure observer's post-lesson reporting of their immediate impressions. There were also interviews with the teacher, collection of student products, a short unstructured survey of the students' responses to the lesson, and other naturalistic records of teacher actions. Analysis of these data involved seeking evidence and effects of teachers making pedagogy explicit, task sequence differentiation, and elements of the activities and interactions that seemed to contribute to the building of a learning community.

The intent of the lesson was for students to use squared paper to gain a stronger sense of area-as-covering to lead towards deriving their own rules or formulae for calculating areas of shapes. There was a range of tasks: in the first the students used grid paper to draw letters using 10 square units, aiming to get them to a stage where they could draw triangles of a given area. In later stages, the students undertook more formal, but open-ended, area investigations. Interim tasks for each stage were offered the necessary experiences to allow the students to reach these points.

The explicit pedagogies

In the lesson documentation, the teachers were offered suggestions about pedagogies to be made explicit, and did this in their own ways. For example, Teacher B presented some directions on a poster (Figure 1), and read through them, as well as giving additional instructions such as:

… even though we won't be talking during the work … if you're sitting next to someone and they have got something different, don't be influenced by that, because there are many, many answers to each of the questions.

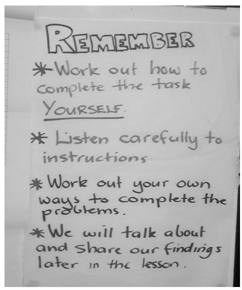

Figure 1: Some explicit pedagogies.

In this project, making aspects of pedagogy explicit has been found to come quite naturally to teachers, has been accommodated into classroom routines readily, and has influenced student actions and learning outcomes (see Sullivan et al., 2003).

The mathematical norms

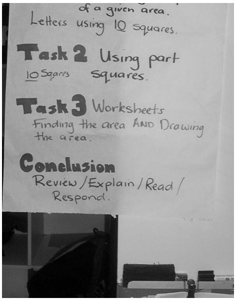

Figure 2: The explicit lesson plan.

The second planning element is the mathematical intent of the lesson. As indicated in the plan prepared by Teacher B (see Figure 2), the lesson was in four parts: Initially students drew letters using 10 square units (see Figure 3), then drew letters of area 10 square units using half squares (see Figure 4 below), then solved conventional area tasks about rectangles and triangles using one set of closed and one of open-ended tasks.

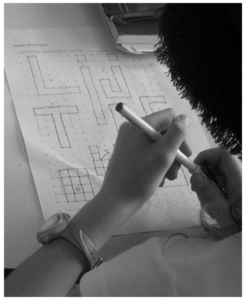

Figure 3: The first task, using squares.

The sequencing of the tasks and awareness of alternative trajectories to meet the ultimate lesson goal was central to the lesson, and is discussed in detail in Mousley, Sullivan, and Zevenbergen (in press). The tasks were structured to be accessible to most students but sufficiently interesting to engage those who quickly grasp the mathematical content. In this case, the possible variations inherent in drawing the letters to the given constraint offered those students who understood the nature of area as counting squares the possibility of creativity as well as some challenge.

The learning community

The class working more or less together on the tasks, as a community, had three aspects. First, all three teachers conducted short whole class reviews after the students had completed each of the respective tasks. Classes B and C worked through each of the lesson stages together and there were no observation notes to indicate that there were students who were unable to participate in the reviews of any of the lesson stages. Key aspects of these reviews were the diversity of student responses and the variety of insights gained into describing and calculating areas. These reviews allowed the teachers to emphasise key mathematical ideas, including those that layed a foundation for the following activity. Both Teachers B and C encouraged students to look at and comment on each other’s work. Teacher A selected the work of particular students and discussed it with the class. Such mid-lesson reviews are already part of the everyday routines of these teachers.

Second, variations to the tasks were proposed to, or prepared by, the teachers. The intent was that if individual students were unable to complete a set task, then a variation could be posed. For example, Teacher A had prepared some sheets of squared paper with one letter of 10 square units already drawn. In the observed lesson, the initial task was introduced but only a few students were able to commence so Teacher A distributed the additional sheet to the others. Having a model to follow was enough to allow them to commence. Teacher A had also prepared some squares cut from card, to allow students to drop back to a more basic entry point, but these were not needed. Likewise for the second task, drawing letters of 10 square units using some half squares, Teacher A had prepared an additional sheet with an illustrative letter. Again, this sheet was given to the students who could not start, and for most this was enough. This teacher had also prepared some cardboard squares and half squares and gave them to those students who still had not commenced the task. These extra activities allowed all students to reach and succeed in the goal task.

We argue that additional supports such as these should be offered discretely. Both Teachers B and C, when offering the additional prompts, made public statements to the class. For example, Teacher B said to the whole class:

> If you are having trouble with the letters, I have some squares here, and you may want to come out and make a letter—and if you still can't get your mind around it, I have some sheets out here that have an example on it, and you can use that if you would like.

Some students did take up the offer as can be seen in Figure 4. Teacher C, however, dealt with the differences by suggesting, "Anybody who is unsure, stay on the floor with me. If at any time you are unsure, come back to me". Both of these approaches are less satisfactory ways of dealing with difference in that they both draw attention to the students experiencing difficulties (which could affect their willingness to seek the assistance needed) or breed dependence.

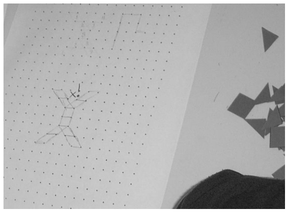

Figure 4: One students' work on the second task.

However, we noted that is not easy to be discrete in classrooms, as was evident with Teacher A. When some students were offered the square and half square counters, most other students insisted they be given the pieces. As it happened, about half the students found it easier to solve the task using the squared paper and the other half found it easier to use the counters. Noticing this, Teacher A then challenged all of the students to use the representation other than the one they were using, and the rest of the lesson was spent doing that. Note that the teacher could not have anticipated that particular students would prefer one or other of the representations and would find the other more difficult, so being both observant and willing to adapt the lesson plan was important. This illustrated how acting on to students' responses can be a key strategy in meeting the needs of the individual students.

Third, extensions were prepared and posed for those students who finish tasks quickly. This is perhaps the most important planning element since students who complete the set work have the effect of moving the lesson along, perhaps before some other students are ready, thereby removing the sense of community. Teachers A and B both posed extensions to students who had completed responses. Interestingly even though Teacher C had posed additional task such as “Choose your most difficult letter and see if you can draw it two different ways”, in the lesson review, she said “It needs more extension work after the first sheet to give me more time to see their work … I was not able to see who needed help”.

We suspect that implementing these task variations is both unfamiliar to teachers and also quite different from the conventional ways of dealing with differences.

SUMMARY

Finding ways to address differences in the backgrounds of students is perhaps the most pressing challenge facing mathematics teachers. The challenge should therefore be at the forefront of the minds of researchers. We are investigating lessons in which teachers make otherwise implicit pedagogies explicit, in which there are carefully sequenced tasks that prompt mathematical exploration and activity, and in which the learning communities are fostered by planning variations to particular tasks to ensure that all students can engage productively and successfully at each stage. It seems that teachers are able to accommodate being explicit about pedagogies within their usual routines, that they can implement appropriate sequences of tasks, they can conduct reviews of student products and whole class mathematical discussions, but that there are a number of aspects of implementing variations to the tasks that seem to be quite different from the conventional teaching routines used by these teachers.

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