# SITUATED OR ABSTRACT: THE EFFECT OF COMBINING CONTEXT AND STRUCTURE ON CONSTRUCTING AN ADDITIVE (PART-PART-WHOLE) SCHEMA

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*This study investigated the development of an additive schema from the perspective of the schema's flexibility in coping with new context and unfamiliar semantic structure. It followed 27 first graders who learned addition and subtraction using an experimental curriculum. The instruction involved a didactical model combining the context of two stories with a structured part-part-whole schema. In addition to collecting data from the whole class, eight children were individually interviewed. The development of their additive structure was examined by comparing the distance between the original stories and the range of transfer problems they could analyze. The findings show that a rich additive structure was constructed, although some difficulties involving dependency on the instructional model were observed.*

Many elementary schools in Israel use a curriculum that is based on introducing mathematical concepts by using structured instructional models that are isomorphic to the mathematical structures. The rationale of this approach is that mathematical concepts are abstract and therefore should be represented by something children can communicate about in natural language (Nesher, 1989). Although there is no argument against this rationale, there are claims that in using such models, for example, teaching addition and subtraction by using Cuisenaire Rods, an expert model is explicitly imposed upon the child (Cobb, Yackel & Wood, 1992).

These structured instructional models together with their rules of operation constitute a Learning System (LS), as termed and described by Nesher (1989). An ideal LS uses an instructional model to define the targeted mathematical objects and mathematical relations and later facilitates a gradual shift from talking about the objects of the model to talking about the mathematical objects they represent.

The instructional model used in the LS is usually a sterile model, i.e. it uses concrete objects such as Cuisenaire Rods (for addition and subtraction), Dienes Blocks (for place value), etc. This model does not usually involve any context. Context and situations that can be mathematized, i.e. mathematically modeled by using the taught mathematical concept, appear later as applications of the LS.

In alternative instructional approaches situations and context come first. Some programs use a problem based curriculum that starts by introducing real world problems or authentic (relevant rather than real) problems. These approaches differ from each other in the amount of structure imposed on the solvers. In some cases children are expected to suggest a variety of solutions using their existing schemas, in others they are given tools to be used in coping with the authentic situations. The Dutch curriculum encourages children to reinvent mathematical models by analyzing situations with the help of tools such as the empty number line. Their theory is described by Gravemeijer and Stephan (2002) terming the stage of analyzing and realizing the structure of the situation as building a model *of* that specific situation, and further on abstracting this structure to a model *for* reasoning mathematically that can be used in mathematizing new situations.

An approach that introduces a mathematical concept with the use of situations risks leading children to a wide range of ideas that might not include the targeted mathematical structures. Even if the right structures are created children might construct a situated model and remain in the *model of* level. On the other hand, a sterile approach might create a sterile structure without meaningful connections resulting with difficulties in identifying the relevancy of the mathematical concept to its potential applications.

Through the years we have tried different combinations of structure and context in an effort to create meaningful structures. Peled and Resnick (1987) used structure and context in designing a train-word, a microword involving zones where carts were added or taken off trains in a way that corresponded simultaneously to mathematical definitions of addition and subtraction and to different semantic categories of word problems. Similarly, in the design of software for kindergarten (Peled, Meron & Hershkovitz, 2000) the designers introduced addition and subtraction using a context that involved the exchange of presents between some characters. The situation created an intrinsic need to act on objects in a way that could be mathematized as addition or subtraction. Although the situations in these examples help children make sense of the structure and learn it more meaningfully, the targeted structure is explicitly imposed.

In this study we introduced addition and subtraction in first grade using stories that lend themselves to an additive mathematization. That is, a representation of the story corresponded naturally to two parts and a whole, and included a rationale for putting the parts together. Thus, the operations were not presented as a sterile model prior to meeting any situations, but were expected to emerge through the children's operations within the situations. In this sense the expected schema development was similar in nature to Gravemeijer's (1997, 1997b) terms of *model of* and *model for* mentioned earlier and also discussed later in more details.

The special role of these contexts or source stories raised the question whether the constructed schema would be limited, enabling applications to problems that are similar in context to the two source stories. That is, whether children's knowledge would be situated. Based on research on analogical thinking (Gick & Holyoak, 1983), we assumed that having two stories would help facilitate schema abstraction. These theories also predicted that problems differ in degree of difficulty to be mapped to the source problem (Gentner & Toupin, 1986). This study investigates the nature of schema children construct in this experimental instruction, and the degree of difficulty of transfer problems.

## METHOD

The study was conducted in first grade in a school with population of an average to slightly less than average socio-economical status. The class was taught by its regular teacher using an experimental curriculum. Of the 27 children undergoing this instruction, 8 children were chosen to be individually interviewed three times through the whole unit of instruction on addition and subtraction. The 8 children included 4 average children, 2 above average and 2 below average, assigned by the teacher according to the interviewer's specifications.

The study consisted of three instruction and evaluation parts. Each part included an instructional unit followed by a whole class questionnaire and by individual interviews with the 8 chosen children.

Each instructional unit began with the teacher telling a story. The story involved a situation of putting parts together and a situation of separating into two parts or taking a part away from the whole. The first story was about a grandfather who has two grandchildren. In some of the situations grandfather sent presents to the children and in others the two children sent presents to him. The second story was about children who live on two islands and travel by boats to school. In some situations the children were going from the two islands to school, and in others they went from school back home.

With the first story the children were asked to model the story (engage in *direct modeling* in Carpenter and Moser's (1984) terms) by moving concrete objects (unit cubes) on a drawn part-part-whole schema, the *Presents' Board*, as depicted in figure 1. For the second story the children had another (similar) board on which they could use unit objects or Cuisenaire Rods.

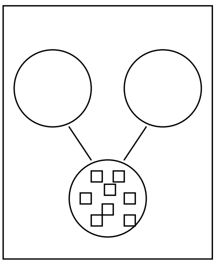

Figure 1: The part-part-whole Presents' Board.

Following the instructional part the whole class worked on a questionnaire that included the completion of part-part-whole schemas involving familiar and unfamiliar situations in the same context (grandfather or boats). The last (third) questionnaire included computational problems as well. In the interviews children were given problems that were similar to the source stories and problems that were different from these stories on the following dimensions: context and roles of characters, semantic structure, and logical structure.

On the basis of an analysis of these three dimensions we built a hypothesized measure of problem difficulty. Our assumption was that problem difficulty would increase when the context changed or when the characters were used in a different role. We assumed that problems with semantic structure that is known to be more difficult (Nesher, Greeno & Riley, 1982) would indeed be more difficult and that problems with an unknown part would be more difficult than problems where the whole is unknown. As we found later, there were cases where the results did not fit with our predictions, some problems with a low difficulty measure were found to be more difficult than expected and vice versa.

An answer was considered as an indication that the student understood the problem structure, if the student solved the problem correctly, retold the problem in a way that made sense, represented it correctly using a number schema (writing the 3 numbers on a part-part-whole template as seen in the example represented in Figure 3) and later also mapped it to a computational expression.

The research purpose was to investigate whether this instructional approach could produce a rich additive schema. The criterion for determining the quality of the schema was the extent of the schema's flexibility as observed by looking at the range of the problems children could understand.

## RESULTS

In order to test the effect of instruction 19 problems of different difficulty measures were given to the 8 children in each of the three interviews. In this paper we focus on the changes that were observed in problem solving throughout the three interviews.

Figure 2 depicts the changes in problem understanding for student #1. The problems were arranged by growing degree of difficulty as determined by the three factors mentioned earlier.

The student's answers were coded according to his understanding. As it turned out, there were some problems that were immediately solved (marked in light gray) other problems took some time to analyze, and in some cases the student constructed a schema and then realized it could not be right and changed it. These solutions were termed "not spontaneous" (marked in medium gray).

As can be seen in figure 2, the more difficult problems were mostly those that were predicted to be difficult using the three factors criterion. In the second interview more problems were spontaneously solved and eventually almost all problems were solved. In the third interview all problems were eventually solved and there were more problems that were spontaneously solved.

| Problem | Difficulty | Interview | | |
|---|---|---|---|---|
| | | 1 | 2 | 3 |
| 1 | 1 | | | |
| 2 | 1 | | | |
| 3 | 2 | | | |
| 4 | 2 | | | |
| 5 | 3 | | | |
| 6 | 3 | | | |
| 7 | 4 | | | |
| 8 | 4 | | | |
| 9 | 4 | | | |
| 10 | 5 | | | |
| 11 | 5 | | | |
| 12 | 5 | | | |
| 13 | 6 | | | |
| 14 | 6 | | | |
| 15 | 6 | | | |
| 16 | 7 | | | |
| 17 | 7 | | | |
| 18 | 8 | | | |
| 19 | 8 | | | |

spontaneously — not spontaneously — not understood

Figure 2: Problem understanding of student #1 through the 3 interviews.

Problem #14 is an example of a problem that was considered to have a degree of difficulty 6, and turned out to be more difficult than expected (it was difficult for the other children as well). The following description shows how student #1 coped with this problem.

The problem: There were 8 red and yellow flowers in the vase. 3 of the flowers in the vase were red. How many yellow flowers were in the vase?

First interview: The student uses the presents' board. He picks up 8 unit cubes and puts them in the place intended to represent a part while saying *8 reds*. Then he puts 3 unit cubes on the other part saying *3 yellows*. He counts them all, makes a counting mistake getting a 10. He then writes a number schema using 3 and 8 as parts and 10 as the whole (as seen in Figure 3). He tells the story: *I had in my vase 8 red flowers and 3 yellow flowers*. The interviewer asks about the 10 and he says: *That's all there is in the vase.* Now he tells the story again: *There are 10 flowers in the vase, 8 reds and 3 yellows.*

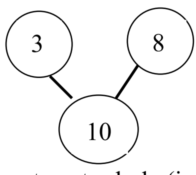


Figure 3: A part-part-whole (incorrect) number schema.

Second interview: The student uses Cuisenaire rods. He takes a rod representing 8 and then puts a rod representing 3 next to it. He gets a rod representing 11 that fits the given length and writes a number schema with 8 and 3 as parts and 11 as the whole. Suddenly he says: *This is wrong*. He changes the numbers in the number schema so that 8 will be the whole and 3 and 5 are the parts. He tells the story: *There were 8 flowers in the vase, 3 reds and then also 5 yellows.* The interviewer asks: *I don't understand. There were 8 flowers and then they put 3 reds and also 5 yellows?* The student answers: *No. There were 3 red flowers and also 5 yellow flowers and together they are 8.*

Third interview: The student puts 8 and 3 rods as parts. He does not complete the rod schema but instead asks to hear the problem again. He then looks at what he constructed and moves the 8 rod to represent the whole. Again he asks to hear the problem and then says: *Ah, the whole is 8.* He completes the rods' schema and then writes a correct number schema.

Similar figures (to figure 2) were drawn for each of the students. Other representations looked at performance over all the children within each interview.

To determine the effect of each of the three factors that were used in predicting difficulty, the average number of problematic solutions was calculated in each interview as a function of the degree of difficulty predicted by that factor. The findings show that the semantic category and the place of unknown predicted correctly the degree of difficulty. The distance in context including change in roles between characters in the stories was not as crucial as expected and the predictions based on it resulted in some incorrectly estimated difficulty rank.

In general, most of the students (7 out of the 8 interviewed students) improved from one interview to the next, and managed to understand most of the problems in the third interview. A large part of the problems was spontaneously understood in the third interview. An interesting observation concerned the spontaneous answers. In looking at the stories that children were asked to compose and sorting them by their spontaneity and structure, it turned out that all spontaneous answers involved a correct additive structure.

## DISCUSSION

Although the part-part-whole schema was introduced in this experimental curriculum in the context of two specific stories, our findings show that the schema constructed by children was not situated. This conclusion is based on observing transfer to more difficult and more distant new problems and interpreting this fact as an indication of the existence of a flexible schema.

Three factors, including context, were used in determining the difficulty of a new problem. The results show that the semantic structure of the problem and the identity of the unknown predicted the degree of problem difficulty better than the problem's distance in context from the source stories. These results have several possible interpretations. We might say that the instructional combination of structure and context resulted in a stronger effect of structure over context. Children were able to focus on the problem's deep structure and were less affected by surface structure (context and roles in the story). However, this interpretation does some injustice to the role of the stories as observed through the study.

In an example from the Dutch curriculum addition and subtraction were taught using a story-context of a Double Decker bus with people getting on and off it. This context was accompanied by an arithmetic rack with two lines of ten beads on which the people (and actions) on the bus could be represented. Gravemeijer et al. (2000) use this example to discuss children's shift from working within the situation, using the beads at first to represent people on the bus to working with the beads on a more abstract level later using the beads to represent the quantitative structure. In their terms, children are involved in an organizing activity and thus the beads' rack changes roles from being a model *of* the situation to becoming a model *for* reasoning that can be used for reasoning about the mathematical relations, a tool for mathematizing other situations.

In our study children used the part-part-whole template in a similar way to the use of the arithmetic rack. During class instruction they used unit cubes on the template to represents the presents that were given by grandfather to his grandchildren. Later they used Cuisenaire Rods to represent the boats that were carrying children from their school. While viewing the cubes and rods as characters in the story they were still engaged in organizing these situations. Further on during their instruction children were asked to map between mathematical expressions and stories in the given context. This stage facilitated the abstraction of the quantitative problem structure.

When word problems with unfamiliar context and new semantic structure were introduced during the interviews, it was possible to observe that some of the children had already shifted to a model *for*. Children who could spontaneously use the number template to represent the quantitative structure of the new problem had probably shifted to thinking about the schema as representing a mathematical structure. Children who took more time to cope with the problems seemed to be using the template as a place for acting first on the objects of the new situation before being able to relate to the mathematical structure. Some children still needed "a period of organization" but this period was becoming shorter in later interviews. The change from one interview to another showed that more problems were solved by more children spontaneously, indicating their shift to a more abstract schema.

We conclude by suggesting that the use of this instructional approach combining context and structure helped children construct an abstracted additive schema. The use of two stories rather than one seemed to facilitate this abstraction, although the shift in performance from the first interview to the next could also be attributed to more instruction. It should be noted that this article reports only on children’s problem solving. In the study we also observed how children solved numerical equations and how they understood them. Some of the children exhibited a need to use objects (cubes or rods) and in some cases relate to the stories in order to solve them.

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