METAPHORS AND CULTURAL MODELS AFFORD COMMUNICATION REPAIRS OF BREAKDOWNS BETWEEN MATHEMATICAL DISCOURSES

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We noticed that when workers try to explain their mathematical practices to inquisitive outsiders, breakdowns in communication arise. We present here an example in which a worker spontaneously uses metaphors and models to facilitate explanation and communication. We analyse these, drawing on Lakoff & Johnson (1999) and Lakoff & Nunez (2000) in substance and theoretical approach. We suggest that metaphors and cultural models ground associations between academic and workplace discourse genres, and point out how sensori-motor groundings of the 'basic' metaphors may afford gesture and image-schema which free discourse from formal mathematical language. In general we see breakdown repairs as being built through cultural models that extend beyond local mathematical genres which situate mathematics within academic or workplace contexts.

INTRODUCTION

This paper will provide theoretical development of the role of metaphors and models in repairing breakdowns in communication, regarded as an essential component in learning and problem solving in general, and mathematical modelling in particular. We build on the work of Lakoff, Johnson, Nunez and Sfard with respect to metaphor and communication, and others such as the Freudenthal Institute with respect to modelling. Our work relates particularly to bridging the gaps between mathematical practices and understanding in Colleges and workplaces (e.g. Williams and Wake, 2002 and Williams, 2003).

We argue that the differences between workplace practices and College practices and discourses can be explained by the different structures of their activity systems, and that breakdown moments arise in dialogues between workers and outsiders because of this. We argue that metaphors and models sometimes have a significant role in helping workers to explain their mathematical practices to researchers and students, thus 'bridging the gap' in meanings and understanding that previous researchers have highlighted (e.g. Williams et al, 2001; Pozzi et al, 1998).

Our research involved developing case studies of mathematical practices across a range of workplaces. Data includes detailed transcripts of conversations between

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workers, teachers and College students as each tried to make sense of, and explain to each other, their understanding of the mathematical practices of the worker. Our analyses follow a multiple case study methodology (Yin, 2002). We will here touch on the data set from one case, just sufficiently for our main purpose which is to develop our theoretical understanding of modelling and metaphor.

Metaphors such as ‘the computer is a servant’ and associated cultural models (the computer ‘thinks’, stores in memory, recalls, etc) are effective as a means of communication as well as a means of thinking ‘to’ oneself. (Holland & Quinn, 1987; Lakoff & Johnson, 1999; Gee, 1996). In the following example, we will look at a case of breakdown in which such an appeal to a cultural model seemed critical in breakdown repair.

THE TIME-LINE MODEL FOR ESTIMATING GAS CONSUMPTION

We illustrate the power of grounded metaphorical models by way of an example, in which a mysterious, and rather complex, spreadsheet formula is explained by its author, an engineer in a power plant who is responsible for estimating the plant’s total daily use of gas based on consumption during part of the working day. A breakdown occurs when the researcher fails to follow the explanation of the times and readings involved, particularly the use of T2 and TIME4 in the formula:

{{{{{“2nd INTEGRATING READING”-“0600 INTEGRATING READING”}+{{{“2nd INTEGRATING READING”} – {“1st INTEGRATING READING”}}/T_2}*TIME4}}/100000}/3.6*CALCV*1000000/29.3071}

The formula uses three meter readings, *A* (0600 INTEGRATING READING) taken at the beginning of the gas day (0600) and the others, *B* (1st INTEGRATING READING) and *C* (2nd INTEGRATING READING), taken a short interval ($t=T_2$) apart just before the estimate is calculated. These are supplied by technicians who complete a data collection form for the engineer. Below, in Figure 1, we re-present the formula in a mathematical genre which we as ‘academic mathematicians’ possibly feel more comfortable with.

gives the actual volume of gas used up to the time the estimate is calculated

$$(C-A)+\left(\frac{C-B}{t}\right)\times T$$

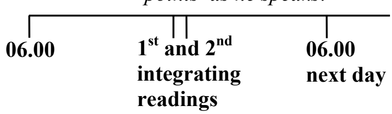
gives an estimate of the rate of gas consumption at the time the estimate is calculated

represents the time remaining of the gas day (TIME4).
Multiplying T by the rate of gas consumption gives an estimate of the gas that will be used during this period.
This is added to the amount already used to give the final estimate for the consumption for the whole gas day.

Fig. 1: Workplace spreadsheet formula re-presented in a mathematical genre.

During discussion with the researcher, Kate, the worker, Dan, makes the construction of the formula clear by recourse to drawing a timeline, on which he marks the instants during the 'gas day' when readings have been taken, and gestures to the intervals between these points on the time-line as time-intervals (e.g. T2 and TIME4) which are used in the construction of his formula.

	Breakdown and inquiry about 'T2'	***Comments***
Kate	Yes. Oh so that *time* that you've got there, that…? Is it T_2?	*T_2 refers to time as in 'interval' not 'point', but Kate is unclear*
Dan	Yes, there's another calculation in there, it gives you T_2	*Control room workers input the times when readings are taken and a spreadsheet formula calculates the time interval, T_2.*
Kate	So T_2: that's the *time*? So is it the time from first to the last, or is it a combined…	*inquiry about T_2: still confused*
Dan	Because you've already got… All I'm interested in, is... Let me draw it out…	*Dan is lost for words... draws the following time-line sketch, marking points as he speaks:*



Dan	The gas day: 0600… reading 1,… reading 2, … end of gas day....	*'gas day' is 0600 – 0600 next day*
Dan	You've got a reading *there (1)*, and you've got a reading *there (2)*… so subtract one from the other, and you know how much you've used *there (3)*.	*Gestures to points and intervals on the line:* *(1) i.e. first 06.00 reading* *(2) i.e. 2nd integrating reading*
	Reading 1 and reading 2, subtract one from the other, you know how much you've used *there (4)*;	*i.e. 'how much gas' used* *(3) i.e. between first 06.00 reading and 2nd integrating reading*
	but you also know the *time* difference between *there* and *there (4)*.	*(4) i.e. between 1st and 2nd integrating readings (T_2)*
Kate	Yes.	

In this 'applied version of the number line, we see:

- the number line metaphor (itself a metaphorical blend);
- life, the 'gas day', is a journey, with source-path-goal;
- time is a 'path' along a line through points in space, and
- instants in time and gas readings are 'points', and intervals between the points are both lapses of time and quantities of gas consumed.

Consequently, every point is fused with (i) an instant in time, (ii) a gas reading, and (iii) the (pair of) numbers or algebraic symbols that represent these. The pair of numbers involved suggests the need for a coordinate pair, i.e. a graph rather than a line. This was in fact introduced as an explanatory model by the teacher later on.

Every *line segment* can be blended (in Lakoff's metaphorical term) or fused (in the semiotic terminology of Werner & Kaplan) with (i) a *time-interval*, (ii) a *quantity of gas* and the (iii) *numbers or algebraic symbols* that represent these.

Gestures (pointing to 'points', waving back and forth at 'intervals') and indexical (pointing, waving) pronouns in the discourse (it, here, there, between) associate the concepts with which they are fused, and implicitly index expressions in the spreadsheet formula. Thus the timeline affords a sensori-motor and associated discursive world of engagement, (a) grounded in the space-time image-schema and narrative of passing through time, and (b) with points and intervals on a line modelling the time and gas consumption involved. This obviates the need to call on formal language such as ‘time interval’ and ‘instant in time’, ‘gas reading’ etc. in favour of pointing gestures which can convey the relevant meanings. Indeed the verbal equivalent of ‘between here (point 1) and there, (point 2)’ might be as complex as “from the time of the first integrating reading to the time of the second integrating reading”, or “the gas consumed between the first and second integrating readings” and indeed the non-verbal gesture may be read in either manner. Thus gestures make communication both easier, more fluent, and perhaps more ambiguous, allowing the interpreter to metaphorically generate meanings initially lacking precision, but affording negotiation and progressive refinement.

Thus, according to Roth (2001) ‘gestures constitute a central feature of human development, knowing and learning across cultures’ (page 365) and he shows how expositions with graphs in a science education context can ‘have both a narrative (iconic gesture) and grounding functions (deitic gestures) connecting the gestural and verbal narratives to the pictorial background’ (p 366).

But Roth’s review suggests the significance of gesture is even deeper than this: there are suggestions in the literature that gestures provide access to another dimension of communication. For instance, when gesture conflicts with the verbal, it usually signifies a transition in meaning or development of understanding, and gesture leads the verbal! In Roth’s own studies, (op cit) the emergence of coherence from ‘muddled’ verbiage in children’s explanations is accompanied by gestural embodiment of relations in advance of their formal, verbal articulation. In sum, gesture can provide a midwife for conception.

If this is the case in the above example, then the number line is surely as important as the midwife’s obstetric instruments. It is a particularly apt tool for the purpose, affording the precision of gesture required to associate the context with the mathematical formula with optimal efficiency.

The number line then we conceptualise as a semiotic, mediating tool through which a formula is associated with a 24-hour time line and the estimation of gas consumption. Its accessibility rests on its status as a ‘cultural model’, widely shared among an educated community which reaches beyond the specialised communities of the workplace, though perhaps only by those sufficiently mathematically prepared to appreciate it. We might call this a mathematical-cultural model. This particular model proves particularly powerful due to its incorporation of metaphorical blendings, within mathematics (space, time, measure and number) and fusions between mathematics (symbols, points, formulae) and the gas day.

CONCLUSION AND DISCUSSION

We conceive of the 'bridging of the gaps' between mathematical practices and discourses (at breakdown moments) as the negotiation of a chain of signs, in the Peircean sense (see also Cobb et al, 2000, and Whitson in Kirschner & Whitson, Eds., 1997). The introduction of new semiotic mediating tools (such as metaphors) can afford 'new' links between signs which result in new chains and interpretants, and hence meaning and understanding. Workers, and perhaps informal 'teachers' and 'explainers' generally, seem to naturally appeal to or reach out to cultural models that can support such semiosis.

At the level of social languages and discourse genres, or Discourses in Gee's (1996) sense, we picture a landscape consisting of:

- workplace language (e.g. the 'gas day');
- workplace (and workplace mathematical) discourse genres (e.g. the spreadsheet formula is written in a 'spreadsheet genre');
- formal academic mathematical genres – e.g. mathematical signs, (e.g. their academic uses as in our 'translation' of the spreadsheet formula above), and mathematical diagrams (e.g. the number line, points and intervals and labels);
- everyday language including cultural models (e.g. metaphor, cultural models);
- gestures, (e.g. pointing to a symbol in a spreadsheet formula, then to an interval of time on the time-line).

The worker's and outsider's discussion helps to constitute a semiotic chain through these domains: a successful conclusion of which may allow the outsider to arrive at an interpretant which is experienced as meaningful to them, (e.g. the formula comes to represent for the researcher a linear extrapolation of gas consumption quantities over time). In such a hypothetical semiotic chain then, a breakdown can occur when the outsider experiences a failure to link: and a missing link may then be supplied by virtue of a mediating chain through a cultural model such as a number-time line.

We hypothesise that such appeals to cultural models may be available to individuals' internal conversations, on the intramental plane, just as they are in interpersonal conversation, i.e. in the interpersonal plane. In the case described above, for instance, it seems likely that the worker made use of the cited models and metaphors in his own personal work practice before the arrival of the researcher, i.e. when writing programs and when developing the formula for estimating gas consumption. However, once developed the spreadsheet functions adequately each time the engineer inputs the appropriate data values, and there is no longer the need to understand the mathematics that underpins the calculations.

On the other hand, the interpersonal conversations may themselves be generative of new chains and meanings. A metaphor 'dawns' in the first instance without one necessarily being fully aware of all its potential for a full-blown analogy. Thus the timeline begins perhaps as a 'bare' line, but then it is marked with various indications of instants and intervals, times and readings... the full implications of the metaphoric blend emerge. In general the analogy may subsequently emerge from a generative process (Schon, 1987) of interpersonal or intrapersonal conversation.

We argue that the time line model (or as Lakoff, Johnson and Nunez would prefer: metaphor) served here as a cultural model for repairing the breakdown caused by the outsiders' lack of familiarity with the particular local discourse genres of the workplace, which combines workplace knowledge and jargon, spreadsheet mathematics and so on in an idiosyncratic way. The outsider, being more familiar with the academic mathematical genre (typified by that of Figure 1), had to 'build a bridge' or 'semiotic chain' between the spreadsheet formula, the workplace task, and academic mathematics. The dialogue, facilitated by the time line, helped her to negotiate this chain of meanings.

We suggest that repairs of communication breakdowns in general might be built through such cultural models, i.e. those that extend beyond the local mathematical genres which situate and embed mathematics within workplace (or academic) contexts. Let us conceptualise mathematical modelling as the process of using such models in solving problems and communicating, and let us build a 'modelling' curriculum around the use of such powerful models in practice.

By studying workplace practices from the perspective of academic mathematics, and especially of the mathematics of College students and teachers, we expose College mathematical practices, and implicitly its curriculum and assessment, to a critical test, or contradiction. We see the research activity then as a potential microcosm of a future, more advanced, mathematical activity and curriculum (Engestrom, 1987). We have therefore begun to see our research into 'workplace mathematical practices' in part as just such a curriculum development.

References

Cobb et al. (2000). (Eds) *Symbolizing and communicating in mathematics classrooms: perspectives on discourse, tools, and instrumental design*. New Jersey: Lawrence Erlbaum Associates.

Engestrom, Y. (1987) *Learning by expanding: an activity-theoretical approach to developmental research,* Orienta-Konsultit, Helsinki.

Gee, J.P. (1996) *Social linguistics and literacies: ideology in discourses,* Taylor & Francis, London.

Holland, D & Quinn, N. (1987). *Cultural models in language and thought*. CUP

Lackoff, K. & Johnson, M. (1999). *Philosophy in the Flesh*. Basic Books.

Lackoff, G. & Nunez, R. (2000) *Where mathematics comes from: how the embodied mind brings mathematics into being*. NY: Basic books.

Pozzi, S., Noss, R. & Hoyles, C. (1998). Tools in Practice, Mathematics in Use. *Educational Studies in Mathematics 36*, 105-122

Roth, W-M, (2001) Gestures: their role in teaching and learning, *Review of Educational Research, 71(3),* 365-392

Schon, D.A. (1987 (2nd reprint 1995) Generative metaphor: a perspective on problem setting in social policy, in Ortony, A. (Ed.) *metaphor and thought* (pp.137-163),CUP.

Werner, H. and Kaplan, B. (1963) *Symbol formation: an organismic-developmental approach to language and the expression of thought,* John Wiley and Sons, New York.

Whitson, J.A. (1997) Cognition as a semiotic process: from situated mediation to critical reflective transcendence. In Whitson, J.A. and Kirshner, D., (Eds.), *Situated cognition: social, semiotic, and psychological perspectives*, (pp.97-149) Lawrence Erlbaum Associates, Mahwah, N.J.

Williams, J.S., Wake G.D. & Boreham, N.C. (2001) College mathematics and workplace practice: an activity theory perspective. *Research in Mathematics Education 3* 69-84

Williams, J.S. & Wake, G.D. (2002). *Understanding workplace practice with College mathematics: a chain of signs mediated by metaphor, discourse genres and models, linking two semiotic practices*. Paper to AERA, New Orleans.

Williams, J.S. (2003). Metaphors and cultural models bridge the gaps in communication between workers and outsiders. *Proceedings of the British Society for Research in Learning Mathematics, 22*(2*)* 79-84.

Yin, R.K.: (2002) 3rd Edition, *Case Study Research: Design and Methods,* SAGE Publications, London