THE NATURE OF SPONTANEITY IN HIGH QUALITY MATHEMATICS LEARNING EXPERIENCES

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Spontaneity has been linked to high quality learning experiences in mathematics (Csikszentmihalyi & Csikszentmihalyi, 1992; Williams, 2002).This paper shows how spontaneity can be identified by attending to the nature of social elements in the process of abstracting (Dreyfus, Hershkowitz, & Schwarz, 2001). This process is elaborated through an illustrative example—a Year 8 Australian male student who scaffolded his learning by attending to images in the classroom that were intended for other purposes. Leon's cognitive processing was not 'observable' (Dreyfus et al., 2001) in classroom dialogue because Leon 'thought alone'. Post-lesson video-stimulated reconstructive interviews facilitated study of Leon's thought processes and extended methodological techniques available to study thinking in classrooms..

INTRODUCTION

The case of Leon, a Year 8 male student, whose exploration culminated in a soft exclamation (when everything finally became clear), is used to elaborate the nature of spontaneity and demonstrate a methodological potential. Cognitive activity that was not 'observable' in classroom interactions (Dreyfus et al., 2001) 'became visible' through post-lesson video-stimulated interviews that were used to probe student reconstruction of thinking.

THEORETICAL FRAMEWORK

M. Csikszentmihalyi (Csikszentmihalyi & Csikszentmihalyi, 1992) identified activities "chosen spontaneously" as associated with a state of high positive affect and total task involvement ('flow'). Flow can occur when a person (or group) spontaneously selects their own challenge that can be met by self-directed development of new skills. Flow experiences specific to mathematical problem solving ('discovering complexity') occur when complex cognitive challenges are overcome during the creation of novel (to the person) concepts (Williams, 2002). Spontaneous development of novel concepts has been studied across a range of topics and age groups. For example, 5 year olds developed concepts of balance and counterbalance during block building activities (Thornton, 1999); 12 to 15 year olds developed 'number-theoretic concepts' during graphical calculator games (Kieran & Guzmàn, 2003), and 16 to 18 year olds developed calculus concepts during collaborative exploration of a complex task (Williams, 2002). To identifying the nature of spontaneity that is critical to improving the quality of mathematics learning, the following research question was posed: How can Leon's case assist in elaborating

the nature of social elements of the process of abstracting that are associated with spontaneity?

RESEARCH DESIGN AND ANALYTICAL TOOLS

This study forms part of a broader study of factors that promote or inhibit autonomous, spontaneous, and creative student thinking associated with high quality cognitive and affective experiences. The study design included three cameras in the classroom simultaneously capturing the actions of the teacher, the whole class, and a pair of focus students over a three-week time interval (see Clarke, 2001 for further detail). A mixed video image with the student pair at center screen and the teacher in the corner was used to stimulate student reconstruction of their thought processes during individual post-lesson interviews. In these interviews, the student controlled the remote, and fast-forwarded through to the parts of the lesson that were important to that student, and talked about what was happening, and what the student thought and felt. The preliminary questions in the interview 'Did you learn anything new today?' and 'What helped you to learn that?' helped to focus the interviewer's probes into what the student had been thinking during the lesson. Video-stimulated interviews with the teacher were undertaken once a week in which she discussed the decisions she had made and what had influenced these decisions. These teacher interviews were triangulated with student interview data, and lesson video data, and also used to elaborate classroom activity that had not been captured by the video cameras.

The present paper draws upon data from three consecutive lessons (L11-13) to describe Leon's thought processes as he abstracted the relationship between rectangles and triangles that enabled him to generalize about areas of triangles. Leon was interviewed after Lesson 12, and briefly again after Lesson 13. The teacher (Mrs Milano) was interviewed after Lesson 13. Some of the interview probes used to stimulate Leon's elaboration of his thoughts included: 'How did that happen?' 'What helped you to make that decision?' Can you tell me more about what you were thinking about?' and 'Can you explain that a little bit more for me?' Two video excerpts (less that 3 minute in total) from the last few minutes of Lesson 12 were selected to illustrate how social elements of the process of abstracting (Dreyfus et al., 2001) can be used to identify spontaneity. The six social elements of the process of abstracting (control, explanation, elaboration, query, agreement, and attention) are elaborated when the results are discussed (later). The particular excerpts were selected because they demonstrated that a process of abstracting can sometimes remain spontaneous when visual images in the classroom form part of the stimuli which enabled the student to structure their investigation.

CONTEXT

In Lesson 11, students found areas of irregular shapes by counting squares. Grids and arrays were then used to develop formulae for areas of rectangles and squares. Early in Lesson 12, students dissented about whether a 10 by 10 square was also a

rectangle. As Mrs Milano asked 'Does a square have four right-angles?' and 'Are opposite sides in a square parallel? Does a square have opposite sides equal?' Leon answered 'yes' softly to himself [Video]. He initially thought she had '… *no clue* ... a square is a square a rectangle is a rectangle. …', then he gradually changed his ideas.

> …I thought (pause) ooh well actually a rectangle (pause) is a lot like a square and a square is a lot like a rectangle [Leon's interview comments about L12, 16:00 Mins]

For the remainder of this paper, 'Mins' has been omitted when the time in Lesson 12 is indicated. Task A was presented twenty minutes into Lesson 12 [denoted as 20:00] when students were asked to find the area of the triangle (1, 2, or 3, see Figure 1) allocated to their pair.
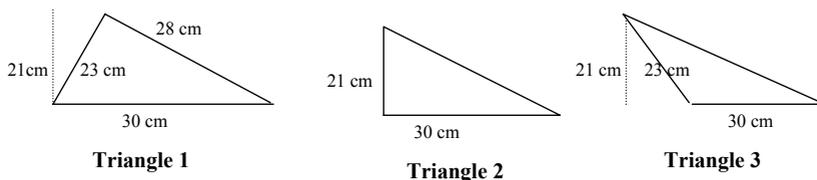

Figure 1. Task A: Colored triangles attached to whiteboard

Most class members (including Leon) lacked prior knowledge of the area of triangle formulae. Leon and Pepe decided to work separately because Leon wanted to think in generalities and Pepe wanted to count squares (in Triangle 1). Leon structured his general exploration by attending simultaneously to the three images on the board. He asked 'Which triangle is easiest?' and rapidly 'recognized' and 'built-with ideas' (Dreyfus et al., 2001):

> … all you have got to do is figure out what a rectangle is that has those two um (pause) lengths- length and width and ... then you can just halve it [Interview report of 23:41].

Leon then focused on off-task activity, and intermittently wondered why Pepe was using a compass. At 36:14 he refocused on his exploration of areas of triangles. During two short time intervals in the next three minutes [36:14-36:22 and 38:21-38:57] Leon considered how to find areas of acute-angled triangles. During the intervening time interval he was involved mostly in off-task activity. By the end of the lesson nine seconds later [39:06], he reported recognizing that 'triangles come in rectangles' and knew a factor of a half was associated with the right-angled triangle case. During Lesson 13, Leon exclaimed as he recognized the generality of the 'half': '... sort of just in my head I pulled it apart and put them together …'. He could have used his right-angled triangle method (applied twice) in developing this insight. He integrated the key attributes of the enclosing rectangles into triangles to find the areas of triangles without explicit reference to rectangles (thereby 'constructing') (Dreyfus et al., 2001). Leon's reflections about Lesson 12 captured the essence of flow experiences in mathematics: '… we really didn't understand … it was a bit of a challenge … when I finally did understand it- it really made me feel good about

myself'. He identified the challenge, concept development, and positive affect. The spontaneity associated with such flow experiences is identified below.

ANALYSIS AND RESULTS

As Leon had identified Lesson 12 as associated with a high quality cognitive and affective experience (above), and that his goal for the lesson had 'changed from *finishing* the work to actually understanding the work', the excerpts used to illustrate spontaneity have been selected to include time just prior to Leon's goal change because it was expected that there would be a rich and diverse nature to the social elements associated with that time. As there was no explicit video evidence of when this goal change occurred, Leon's interview descriptions of this change process were used to identify the relevant video excerpts. Leon reported that preceding the goal change, he '… got different methods in … [his] head of working it out'. He had also described the types of occasions when such goal changes generally occurred for him.

> ... when you look around the classroom and see how everyone else is doing it and you are doing a it a completely different way- … and you think ooh! [soft] maybe my method isn't the best and … you think about *everyone's ...* and then you think about your own and they all sort of piece together and you just sort of go oh! and it pops into your head.

The above evidence suggested Leon's goal change was preceded by thoughts about at least one inelegant method before a better direction of exploration became apparent. It also suggested that visual images produced by other students formed a part of his synthesis of ideas. An enriched transcript (Table 2) was developed to display the evidence used to identify when Leon's goal change occurred. Each row of the transcript dialogue [Columns 5-7; Leon, Pepe, other participants] displays multi-source data that appeared to relate to the same 'instant' in time [Column 1]. The other columns display Leon's interview comments [Column 2], Leon's images sketched in interview [Column 3], and visual classroom stimuli Leon attended to [Column 4]. Where appropriate, body language was included in the dialogue columns. Due to the richness of data available, decisions were made to include only dialogue associated with identifying Leon's development of ideas, and social elements associated with that process. Dotted horizontal lines in Table 2 indicate omitted intervals of time. All of the dialogue included was associated with small group interactions that were either between: Leon and Mrs Milano [36:14]; the pair on Leon's right, Leon, and Mrs Milano [38:21]; or Leon and Pepe (on Leon's left) [38:56] (see Table 2).

Analysis was undertaken to find when Leon developed his inelegant method. This method was described in Leon's interview as he reported reflecting on Pepe counting squares: '… I thought … you could do it quicker … figure out what the area would be fully and then halve it …' [Table 2, 38:56]. When asked to explain, Leon sketched two juxtaposed acute-angled triangles [See Table 2, Columns 2-3] and stated that the irregular shape in his sketch should have been a parallelogram; that the triangles should have the same 'length' and 'width' (the terms side length and height were confused until 'base' and 'height' were introduced in Lesson 13).

Time in L12	Interview reconstruction: Leon	Interview sketch	Lesson Stimuli	Leon: Dialogue [Body Language]	Pepe	Others
36:14-36:22	… I thought I could just figure out what it would be if it were a parallelogram and then halve it … figure out what it would be if it was four because you could just trial whatever it was if it was four. … I understood it- I didn't understand it then I understood it then I didn't understand it.	Asked to explain more, Leon prefaced his sketch with 'Um I … think it was … like that … [drew 2 triangles] should have the same length and width ..'		Leon: [to T] I think I know. [Nodded] Except if I was doing Triangle 2-2's the easiest one there. [Leon does not respond to Pepe's actions]	Pepe: Bip bip [drove a pen over Leon's arm]	T: [to Leon] Do you?
38:21-38:57	He's [Pepe] drawn up the grid and … I thought … you could do it quicker … figure out what the area would be fully and then halve it [see 36:14 above]. … people were drawing the actual rectangles around it- I don't know whether they knew they were coming from rectangles … the way they were drawing it made it look like they did … made me think about it		Elina's and Serina's Pepe's page	Leon: [to T] Aren't they silly? There's so many other shorter ways. [to Pepe] [Laughs] You are doing a great job Pepe.		T: [to Elina and Serina] … You've resorted to drawing the grid?

Table 2. Enriched transcript identifying intervals in the lesson associated with Leon's goal change

This suggested Leon may have partially developed an analytical argument rather than relied solely on perceptual images to recognize the shape of the figure formed (see Dreyfus (1994)). Leon may have selected this process of juxtaposition and informal consideration of properties of the shape formed because he had used it earlier. Leon's intense interest in rectangle properties just prior to his exploration is consistent with this conjecture.

Leon's reconstruction of his thinking after the parallelogram was formed provided inconclusive evidence about his method (which nevertheless appeared inelegant)

> … figure out what it would be if it were a parallelogram and then halve it … figure out what it would be if it was four because you could just trial whatever it was if it was four

When asked to provide further detail, Leon demonstrated the fragility of his ideas: '… I understood it- I didn't understand it then I understood it then I didn't understand it'. Leon's use of 'four' (above) is not consistent with shifting a triangle to the opposite end in Figure 2 to form a rectangle, but is consistent with cutting the parallelogram into four (see dotted lines in Figure 2) to make four right-angled triangles (and find their areas by repeated use of his right-angled triangle method). He might have used 'trial' as checking empirically, or repeating a process.
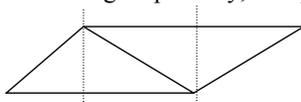

Figure 2. Interpretation of Leon's strategy for finding area of parallelogram

Leon seemed to have been developing this method when he told Mrs Milano he 'knew' and that Triangle 2 was easiest [Table 2, 36:14]. This conjecture is supported in several ways: (a) Leon's method for Triangle 2 is much simpler than the method above; (b) Leon was unaware of the pen Pepe ran over his arm at 36:15 which suggested Leon was involved in his thoughts; (c) Leon participated in off-task activity [36:22-38:21] so was unlikely to have developed the method then; and (d) Leon focused on the grid method for the right-angled triangle [38:21-38:38], then on off-task interactions until 38:56 when we know he was aware of his inelegant method.

As Leon developed his inelegant method first, and did not report being aware that 'triangles come in rectangles' at 38:56, it appears Leon changed direction some time between 38:56 and the end of the lesson (10 seconds later). The fragility of Leon's ideas at 38:56 suggested he could have already had thoughts about more than one approach (that had not yet crystallized). If so, he could already have seen the visual images that contributed to his change in direction. What were these images on student worksheets? And how did they contribute to his change in direction? What was Leon looking at prior to 38:56? During the lesson, Leon had frequently scanned the student activity in the classroom; mainly to find female students within audible range to tease. This activity would also have enabled Leon to see at least five of the worksheets of adjacent pairs, and we know he focused on the worksheets on either

side of him in the minute prior to 38:56. Mathematical explanations appropriate to Leon's exploration were not explicit in the images students produced because pairs were counting squares or finding Triangle 2 using a rectangle [Video, Mrs Milan's Interview]. Leon must have attended to the rectangles students had produced to contain their grids [Table 2, 38:21, Columns 2, 4], and reflected on his right-angled triangle method to recognize that 'triangles come in rectangles'. This provided the impetus for Leon's change in direction, which culminated in his insight in Lesson 13. The spontaneity of Leon's abstracting process during Lesson 12 is now discussed.

DISCUSSION AND CONCLUSIONS

When indicators of spontaneity occur in the absence of indicators of lack of spontaneity, the abstracting process is seen to be spontaneous. The presence of one or more indicator of lack of spontaneity indicates the process is no longer spontaneous. Social elements are examined to determine their nature; firstly whether Leon responded to social elements associated with an external source or to his own activity (for the purpose of this paper called 'external' and 'internal' social elements respectively). Internal social elements included those observable in video data, and those reconstructed in interview. Internal social elements indicate spontaneity (self-directed activity) if no external social elements associated with lack of spontaneity are also present during the exploration (see below).

Examining External Social Elements and Internal Social Elements

External social elements can be associated with lack of spontaneity; they can contain mathematical explanations or elaborations, or control of the direction of exploration. External social elements in the video excerpts included only the two queries from Mrs Milano. Mrs Milano's first query 'Did you?' [≈36:16] led to Leon's elaboration of his thinking. This query did not contribute mathematical information but (like the interview probes) encouraged Leon to express ideas. In the second query [38:21], the use of 'resorted' suggested a more appropriate method existed thus providing mathematical information (that Leon already knew). The second query also focused attention on the grid (where mathematical information was not explicit in the image). As none of the external social elements provided mathematical information to Leon, controlled his direction of exploration, confirmed his direction, or its correctness, or the attainment of closure, Leon's process of abstracting appeared to be spontaneous. (Task A did not control Leon's direction because he did not respond to this control but used the stimuli from Task A differently). None of the internal social elements to which Leon responded were traceable to earlier external elements occurring *during* the abstracting process. Leon structured his exploration with his own queries. He used methods, ideas, and strategies he had developed earlier (e.g., right-angled triangle method, juxtaposition of triangles, properties of shapes, and areas of rectangles). Some of these ideas were traceable to external explanation (area of rectangle), and external attention (considering properties of shapes) *prior to* Leon's exploration. For this reason, these external social elements did not indicate a lack of

spontaneity because Leon possessed cognitive artifacts associated with these ideas at the start of his exploration, and recognized their appropriateness for *himself*. Leon's attention to external stimuli was internal focusing of attention on the mathematics implicit in the images.

Conclusion

Criteria developed to elaborate spontaneity (above) require further refinement and elaboration through their application to other diverse cases where the quality of the learning experience suggests spontaneity might exist. Perhaps, this paper's focus on the benefits of spontaneity might stimulate such further research. Of particular interest is Leon's self-focusing of attention on visual stimuli intended for other purposes. Further research could identify stimuli that can be generated during exploratory activity and used idiosyncratically by students who attend to the mathematics implicit within them. Images identified as inadequate perceptual arguments that require a rigorous mathematical argument to support intuition (Dreyfus, 1994) may stimulate thought about the types of images in which the mathematics is not explicit.

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