IS THEORETICAL THINKING NECESSARY IN LINEAR ALGEBRA PROOFS?

Georgeana Bobos

Concordia University, Montreal, Canada

My research took place within the context of a larger study aimed at investigating the potential of weekly quizzes in developing students' theoretical thinking in an undergraduate linear algebra course. We looked at how students prove in linear algebra, as we first believed that this activity would necessarily engage students with theoretical thinking.

Our data consisted of students' solutions to ten quizzes. We have based our analyses of the data on an assumed correspondence between Sierpinska's model of theoretical thinking (2000) and Harel and Sowder's (1996) classification of proof schemes. For example, Sierpinska postulates that theoretical thinking ("TT" in the sequel) is *systemic*. This, in particular, means that, in TT, the meaning of concepts is defined by their relations to other concepts, not by reference to concrete objects and actions. Thus, in TT, proving a general property requires making connections between concepts based on their definitions and on theorems, and not just on reference to concrete examples. Thus systemic approach to proving engages *analytic proof schemes* in Harel and Sowder's sense, while reference to concrete examples engages *empirical proof schemes.* But, our data revealed that the more advanced students in our study were perfectly capable of writing mathematically correct proofs, as if they used analytic proof schemes, while not engaging in theoretical thinking. They seemed to have mastered a discourse or a "genre" on a superficial level, but not the underlying intellectual attitude.

We then started thinking that perhaps the reason for this to be possible is in the nature of the tasks. The quiz questions were designed to foster theoretical thinking, but they did not entirely fulfil this expectation. Therefore we are currently undertaking a research path concerned precisely with the design of tasks in linear algebra that would make better use of students' potential to think theoretically and have them engaged in structural axiomatic and axiomatizing proof schemes (in the sense of Harel and Sowder, 1996).

References:

Harel, G. und Sowder, L.:1996, *Students' proof schemes.* Manuscript. Purdue University and San Diego State University.

Sierpinska, A.: 2000, 'On some aspects of students' thinking in linear algebra', in J.-L. Dorier (ed.) *On the Teaching of Linear Algebra,* Kluwer Academic Publishers, Dortrecht/ Boston / London, pp. 209-246.