957-26-48 **Peter A. Loeb*** (`loeb@math.uiuc.edu`), Department of Mathematics, University of Illinois, 1409 West Green Street, Urbana, IL 61801, and **Erik Talvila** (`etalvila@math.ualberta.ca`), Department of Mathematical Sciences, University of Alberta, Edmonton, Alberta T6G 2E2, Canada. *Covering Theorems and Lebesgue Integration.*

We use the Morse Covering Theorem to show how the Lebesgue integral can be obtained as a Riemann sum. That is, Let X be a finite dimensional normed space; let μ be a Radon measure on X and let $\Omega \subseteq X$ be a μ-measurable set. For $\lambda \geq 1$, a μ-measurable set $S_\lambda(a) \subseteq X$ is a λ-Morse set with tag $a \in S_\lambda(a)$ if there is $r > 0$ such that $B(a,r) \subseteq S_\lambda(a) \subseteq B(a,\lambda r)$ and $S_\lambda(a)$ is starlike with respect to all points in the closed ball $B(a,r)$. Given a gauge $\delta : \Omega \to (0,1]$ we say $S_\lambda(a)$ is δ-fine if $B(a,\lambda) \subset B(a,\delta(a))$. If $f \geq 0$ is a μ-measurable function on Ω then $\int_\Omega f\, d\mu = F \in \mathbb{R}$ if and only if for some $\lambda \geq 1$ and all $\varepsilon > 0$ there is a gauge function δ so that $|\sum_n f(x_n)\,\mu(S(x_n)) - F| < \varepsilon$ for all sequences of disjoint λ-Morse sets that are δ-fine and cover all but a μ-null subset of Ω. This procedure can be applied separately to the positive and negative parts of a real-valued function on Ω. (Received June 12, 2000)