



TORIC KÄHLER METRICS: COHOMOGENEITY ONE EXAMPLES OF CONSTANT SCALAR CURVATURE IN ACTION-ANGLE COORDINATES

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Abstract. In these notes of the mini-course given by the author at the XI-th Conference on Geometry, Integrability and Quantization, Varna, Bulgaria, 2009 after the introduction of the toric Kähler geometry, we present Calabi's family of $U(n)$ -invariant extremal Kähler metrics in symplectic action-angle coordinates and show that it actually contains, as particular cases, many interesting cohomogeneity one examples of constant scalar curvature.

CONTENTS

1. Introduction	12
2. Toric Symplectic Manifolds	13
2.1. Symplectic Manifolds	14
2.2. Symplectomorphisms and Darboux's Theorem	15
2.3. Symplectic and Hamiltonian Vector Fields	16
2.4. Hamiltonian Torus Actions	16
2.5. Toric Symplectic Manifolds	18
2.6. Classification Theorem and Action-Angle Coordinates	19
3. Toric Kähler Metrics	21
3.1. Linear Compatible Complex Structures	21
3.2. Toric Compatible Complex Structures	23
3.3. Local Form of Toric Compatible Complex Structures	24
3.4. Symplectic Potentials for Compact Toric Symplectic Manifolds	26

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