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INEQUALITIES AMONG THE NUMBER OF THE GENERATORS AND RELATIONS OF A KÄHLER GROUP

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Abstract. The present note announces some inequalities on the number of the generators and relations of a Kähler group $\pi_1(X)$, involving the irregularity $q(X)$, the Albanese dimension $a(X)$ and the Albanese genera $g_k(X)$, $1 \leq k \leq a(X)$, of the corresponding compact Kähler manifold X. The principal ideas for their derivation are outlined and the proofs are postponed to be published elsewhere.

Let X be an irregular compact Kähler manifold, i. e., with an irregularity $q = q(X) := \dim_{\mathbb{C}} H^{1,0}(X) > 0$. The **Albanese variety** $\mathrm{Alb}(X) = H^{1,0}(X)^*/H_1(X,\mathbb{Z})_{\text{free}}$ admits a holomorphic Albanese map $\mathrm{alb}_X \colon X \to \mathrm{Alb}(X)$, given by integration $\mathrm{alb}_X(x)(\omega) := \int_{x_0}^{x} \omega$ of holomorphic $(1,0)$-forms $\omega \in H^{1,0}(X)$ from a base point $x_0 \in X$ to $x \in X$. The complex rank of the Albanese map alb_X is called an **Albanese dimension** $a = a(X)$ of X.

A compact Kähler manifold Y is said to be Albanese general if $\dim_{\mathbb{C}} Y = a(Y) < q(Y)$. The surjective holomorphic maps $f_k \colon X \to Y_k$ of a compact Kähler manifold X onto Albanese general Y_k are referred to as Albanese general k-fibrations of X. The maximum irregularity $q(Y_k)$ of a base Y_k of an Albanese general k-fibration $f_k \colon X \to Y_k$ is called k-th Albanese genus of X and denoted by $g_k = g_k(X)$. The present note states lower bounds on the Betti numbers $b_i(\pi_1(X)) := rk_{\mathbb{Z}} H^i(\pi_1(X), \mathbb{Z})$ of the fundamental group $\pi_1(X)$, in terms of the irregularity $q(X)$, the Albanese dimension $a(X)$ and the Albanese genera $g_k(X)$, $1 \leq k \leq a(X)$.

On the other hand, $b_i(\pi_1(X))$ are estimated above by the number of the generators s and the number of the relations r of $\pi_1(X)$ and, eventually, by the irregularity $q(X)$, exploiting to this end few abstract results on the group cohomologies.