

A NEW FORMULA FOR THE EXPONENTS OF THE GENERATORS OF THE LORENTZ GROUP

GEORGI K. DIMITROV and IVAÏLO M. MLADENOV

*Institute of Biophysics, Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 21, 1113 Sofia, Bulgaria*

Abstract. Here we derive a new formula for the exponent of an arbitrary matrix in the Lie algebra of the Lorentz group. Our considerations are based on the fact that for each constant electromagnetic field there exist an inertial system in which one can easily solve the respective Lorentz equation and therefore to find the explicit formulas describing the trajectories of the charged particles in this field.

1. Introduction

Many mathematical models of processes in Physics, Biology and Chemistry are based on systems of linear, ordinary differential equations with constant coefficients. For example, by rewriting the classical Lorentz force law equation in the relativistic form (see [1], [3] and [7] for more details about this issue), the motion of a charged particle in a constant electromagnetic field can be described by a system of four linear differential equations – the so called **Lorentz equations**

$$\frac{dU^\alpha}{d\tau} = a\mathcal{F}_\beta^\alpha U^\beta, \quad \alpha, \beta = x, y, z, t. \quad (1)$$

Here and below U denotes the particle four-velocity (column) vector with respect to the fixed inertial system

$$U = {}^t(U^x, U^y, U^z, U^t), \quad {}^t = \text{transpose} \quad (2)$$

with

$$U^\gamma = \frac{d\gamma}{d\tau} \quad \gamma = x, y, z \quad \text{and} \quad U^t = \frac{E}{m} \quad (3)$$

being its space, respectively time components, τ is the proper time on the world line, while E is the particle energy, m is its mass at rest and a summation over