D-MATH
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Algebra II

FS 2014

Exercise sheet 10

RELATING ROOTS AND COEFFICIENTS OF A POLYNOMIAL :
ELEMENTARY SYMMETRIC POLYNOMIALS AND DISCRIMINANT

1. Solve the following system in $\mathbb{C}$

$$\begin{cases} z_1 + z_2 + z_3 & = 1 \\ z_1 z_2 z_3 & = 1 \\ |z_1| = |z_2| = |z_3| & = 1. \end{cases}$$

2. Consider $f(x) = x^3 - 2x + 5$ and denote $\alpha_1, \alpha_2, \alpha_3$ its complex roots.

   (a) Compute $\alpha_1^4 + \alpha_2^4 + \alpha_3^4$.

   (b) Exhibit a polynomial $p(x) \in \mathbb{Z}[x]$ of degree 3 and roots α_1^2, α_2^2, α_3^2.

3. Let $w_k = u_1^k + \cdots + u_n^k$.

   (a) Prove Newton identities :

$$w_k - s_1 w_{k-1} + \cdots \pm s_{k-1} w_1 \mp k s_k = 0.$$

   (b) Do $w_1, \ldots, w_n$ generate the ring of symmetric functions ?

4. Let $f(x) \in \mathbb{R}[x]$ be a monic polynomial of degree n, with roots $\alpha_1, \ldots, \alpha_n$.

   (a) Let N be the number of real roots of f. Show that

$$\begin{cases} N \equiv n \bmod 4 & \text{if } D(f) > 0 \\ N \equiv n - 2 \bmod 4 & \text{if } D(f) < 0. \end{cases}$$

   (b) How many real roots can $x^3 + px + q$ have ?