D-MATH
Prof. Brent Doran

Algebra II

FS 2014

Exercise set 9

Splitting fields, finite fields

1. Let F be a field of characteristic zero, and let g be an irreducible polynomial that is a common divisor of f and f'. Prove that g^2 divides f.

2. Let $\mathbb{F}$ denote a finite field. Prove that $\mathbb{F}$ has p^r elements, for some prime $p > 1$ and positive integer r.

3. Let K denote the splitting field of a polynomial $f(x) \in F[x]$ of degree d. Prove that $[K : F]$ divides $d!$.

4. Factor $x^9 - x$ and $x^{27} - x$ in $\mathbb{F}_3$.

5. Let $\mathbb{F}$ be a field of characteristic $p \neq 0, 3$. Show that, if α is a zero of $f(x) = x^p - x + 3$ in an extension field of $\mathbb{F}$, then $f(x)$ has p distinct zeroes in $\mathbb{F}(\alpha)$.

6. Let F denote a field, p a prime and take $a \in F$ such that a is not a p^{th} power. Show that $x^p - a$ is irreducible over F.