Exercise Sheet 1

Exercise 1

For $n \in \mathbb{N}$, consider the Riemannian manifold $\mathbb{S}^n = (S^n, g_{\mathrm{eucl}})$. Given $p \in S^n$, provide an explicit formula for the geodesic symmetry about p.

Compare this to the case of $\mathbb{E}^n$. *What does the formula look like for* $\mathbb{H}^n$*?*

Exercise 2

For $n \in \mathbb{N}$, consider the manifold

$$\mathrm{Sym}_1^+(n) := \{X \in M_{n,n}(\mathbb{R}) \mid X^T = X,\ X \gg 0,\ \det X = 1\}.$$

Define a Riemannian metric on $\mathrm{Sym}_1^+(n)$ which turns it into a symmetric space.

Exercise 3

Let V be a finite-dimensional real vector space and let $A \in \mathrm{GL}(V)$ be such that $A^2 = \mathrm{Id}$. Show that A is diagonalizable with eigenvalues $\lambda \in \{-1, 1\}$.

Recall the case $A = D_p s_p \in \mathrm{GL}(T_p M)$ *where* s_p *is the geodesic symmetry of a locally symmetric space* M *about* $p \in M$.