NM SS 2011

A1

a) richtig.

b) falsch (2. Zeile).

c) ~~richtig (zB bei dünn-besetzten Matrizen).~~

d) falsch,

$\underset{10^{-3}}{\Delta a} = \hat{a} - a \Rightarrow a = 1-10^{-3}$, $\underset{0.5}{\Delta b} = \hat{b} - b \Rightarrow b = 1.5$, Skript p. 10: $\Delta(a \cdot b) = a\Delta b + b\Delta a = 0.501$.

e) richtig,

$F(x) = \frac{1}{5}(\sin x + \cos x) \; : \; x \in [-1, 1]$; $F'(x) = \frac{1}{5}(\cos x - \sin x) < 1 \Rightarrow$ Lipschitz

f) richtig,

$f(z) = z^2 \Rightarrow f'(z) = 2z$.

symm. Diff. Quot.: $f'(z) \approx \frac{f(z+\Delta z) - f(z-\Delta z)}{2\Delta z} = \frac{z^2 + 2z\Delta z + \Delta z^2 - (z^2 - 2z\Delta z + \Delta z^2)}{2\Delta z}$

$= 2z = f'(z)$.

) falsch,

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \Rightarrow \begin{aligned} \dot{x}_1 &= x_1 \\ \dot{x}_2 &= x_2 \\ \dot{x}_3 &= x_1 + x_2 + 5x_3 \end{aligned}$$

) falsch ($j \geq i$).

A2

a) (Skript p. 93)

$h_0 = 2$

$S_0 = \frac{1}{2}(0 + 2^5) = \frac{1}{2} 2^5, \quad T_0 = \frac{1}{2} 2^6, \quad N_0 = 1;$

$\Rightarrow$ **1. Schritt:** $n = 0$

$$h_1 = \frac{h_0}{2} = \frac{2}{2}$$

$$S_1 = S_0 + \sum_{j=1}^{N_0} f(a + (2j-1)h_1) = \frac{1}{2} 2^5 + \left(\frac{2}{2}\right)^5 = 2^5\left(\frac{1}{2} + \frac{1}{2^5}\right)$$

$$T_1 = h_1 S_1 = \frac{2^6}{2}\left(\frac{1}{2} + \frac{1}{2^5}\right), \quad N_1 = 2.$$

2. Schritt: $n = 1$

$$h_2 = \frac{h_1}{2} = \frac{2}{4}$$

$$S_2 = S_1 + \sum_{j=1}^{N_1} f(a + (2j-1)h_2) = 2^5\left(\frac{1}{2} + \frac{1}{2^5}\right) + \left(\frac{2}{4}\right)^5 + \left(\frac{3 2}{4}\right)^5$$

$$= 2^5\left(\frac{1}{2} + \frac{1}{2^5} + \frac{1}{2^{10}} + \frac{3^5}{2^{10}}\right)$$

$$T_2 = h_2 \cdot S_2 = \frac{2^6}{4}\left(\frac{1}{2} + \frac{1}{2^5} + \frac{1}{2^{10}} + \frac{3^5}{2^{10}}\right) = \underline{\underline{2^6 \cdot 0.1924}}$$

b) Trapezmethode $\hat{=}$ stkw. lin. Interpolation. $2 \cdot x$ ist lin. Fkt. $\Rightarrow$ abs. Fehler hier $\underline{\underline{'0'}}$

c) Genauigkeit: $n = 4 \Rightarrow 2n - 1 \underline{= 7}$, da Gaussquadratur.

$$\int_{-1}^{1} x^8 dx \approx \sum_{j=1}^{4} w_j f(x_j) = 0.35 \cdot (-0.86)^8 + 0.65 \cdot (-0.34)^8 + 0.65 \cdot (0.34)^8 + 0.35 (0.86)^8$$

$$= 0.2097$$

A3

a) $n = 2$ (Skript p. 70)

x	f	
0	1	$= P_0$
1	2	$= P_1$
2	4	$= P_2$

$P_{01} = \frac{\frac{1}{2} - 1}{0 - 1}(1 - 2) + 2 = 1.5$

$P_{12} = \frac{\frac{1}{2} - 2}{2 - 4}(2 - 4) + 4 = 2.5$

$P_{012} = 2.5 + \frac{\frac{1}{2} - 2}{0 - 2}(1.5 - 2.5) = \underline{\underline{1.75}}$

b) Absoluter Approximationsfehler: $e^x - (1 + x)$

$$\frac{d}{dx}\left(e^x - (1 + x)\right) \stackrel{!}{=} 0 \Rightarrow \text{Gleichung: } \underbrace{e^x - 1}_{f(x)} \stackrel{!}{=} 0$$

mit Newton: $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} \leftarrow f'(x) = e^x.$

1. Schritt: $x_1 = 1.5 - \frac{e^{1.5} - 1}{e^{1.5}} = 0.7231$

2 Schritt: $x_2 = 0.7231 - \frac{e^{0.7231} - 1}{e^{0.7231}} = \underline{\underline{0.2083}}$

(analytisch klar: Approximationsfehler $\equiv 0$ bei $x = 0$)

A4

a) $F = a t^2 + b \exp(t) + \underbrace{\exp(c)}_{\tilde{c}} \cdot t$

Gleichungen:

$$\left.\begin{array}{lll} t=0: & 0 \cdot a + b + 0 \cdot \tilde{c} - 0.5 \\ t=1: & 1 \cdot a + e^1 \cdot b + 1 \cdot \tilde{c} - 1 \\ t=2: & 4 \cdot a + e^2 \cdot b + 2 \cdot \tilde{c} - 5 \\ t=3: & 9 \cdot a + e^3 \cdot b + 3 \cdot \tilde{c} - 100 \end{array}\right\} \longrightarrow \text{min. (kleinste Quadrate)}$$

→ Matrix:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & e^1 & 1 \\ 4 & e^2 & 2 \\ 9 & e^3 & 3 \end{pmatrix}$$

b) Skript p. 63 : $y_i = \frac{1}{s_i} (U^T)^{(i)} \underline{c}$, $i = 1, 2$

$$x = V y$$

$$y_1 = \frac{1}{2} \cdot \left(-\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, 0\right) \cdot \begin{pmatrix} \sqrt{2} \\ 2\sqrt{2} \\ 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix} = -2$$

$$y_2 = 1 \cdot \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0\right) \cdot \begin{pmatrix} \sqrt{2} \\ 2\sqrt{2} \\ 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix} = 2$$

$$x = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} -2 \\ -2 \end{pmatrix}}}$$

A5

a) Skript p. 119

$$x_1^{n+1} = x_1^n + \frac{h}{2}\left(x_1^n + \exp(x_2^n) + x_1^{n+1} + \exp\left(x_2^{n+1}\right)\right)$$

$$x_2^{n+1} = x_2^n + \frac{h}{2}\left((x_1^n)^2 + 3x_2^n + (x_1^{n+1})^2 + 3x_2^{n+1}\right)$$

Lösen z. B. mit Newtonverfahren.

b) Verwende $f(t,x) = \lambda x$

$$\Rightarrow \quad k_1 = \lambda x$$

$$k_2 = \lambda\left(x + \frac{h}{4}k_1 + \frac{3h}{4}k_2\right) = \lambda\left(x + \frac{h}{4}\lambda x + \frac{3h}{4}k_2\right)$$

$$\Rightarrow \quad k_2 = \frac{\lambda\left(x + \frac{h}{4}\lambda x\right)}{\left(1 - \frac{3h}{4}\lambda\right)}$$

$$\bar{x} = x + \frac{h}{2}k_1 + \frac{h}{2}k_2 = x + \frac{h}{2}\lambda x + \frac{h}{2}\frac{\lambda\left(x + \frac{h}{4}\lambda x\right)}{\left(1 - \frac{3h}{4}\lambda\right)}$$

$\lambda \cdot h := z$

$$\bar{x} = x\underbrace{\left(1 + \frac{z}{2} + \frac{z}{2}\cdot\frac{1}{\left(1-\frac{3}{4}z\right)} + \frac{z}{2}\cdot\frac{z}{4}\cdot\frac{1}{\left(1-\frac{3}{4}z\right)}\right)}_{\text{Stabilitätsfunktion}} = x\underbrace{\left(1 + \frac{z}{2} + \frac{z}{2-\frac{3}{2}z}\left(1+\frac{z}{4}\right)\right)}_{''}$$

A6

NM SS 2011

a)

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a = 2;

b = 3;
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f = @(x,y) 2x*y ;

c = WS (a, b, f)
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b)

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A = [3 2 1; 1 1 1; 3 4 2];
b = [1; 2; 3;] ;
x = A \ b
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c)

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q = (x >= 0) - (x < 0)
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