A1:

a) richtig

b) falsch

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c) falsch (Matrixstruktur von $D + L$)

d) falsch,

$$\delta(a-b) = \frac{a}{a-b}\,\frac{\Delta a}{a} - \frac{b}{a-b}\cdot\frac{\Delta b}{b} \quad ; \quad \begin{array}{l} 10^{-3} = 1-a \Rightarrow a = 1-10^{-3} \\ 10^{-2} = 0.9-b \Rightarrow b = 0.9-10^{-2} \end{array} \Rightarrow \delta(a-b) = -8\%$$

e) richtig,

$$F(x) = \frac{1}{5}\cos^2(x), \quad F(x) \in [0,1]. \quad F'(x) = \frac{2}{5}\cos(x)\sin(x) < 1 \Rightarrow \text{Lipschitz}$$

f) richtig,

$$f'(z) \approx \frac{f(z+\Delta z) - f(z)}{\Delta z} = \frac{\frac{1}{2}\left(z^2 + 2z\Delta z + \Delta z^2\right) - \frac{1}{2}z^2}{\Delta z} = \underbrace{z}_{f'(z)} + \frac{\Delta z}{2}$$

g) richtig

) falsch

A2

NM WS2012

a)

Einsetzen: $\frac{\lambda}{8}\cdot\left(0 + 3\cdot\left(\lambda/3\right)^2 + 3\cdot\left(2/3\,\lambda\right)^2 + \lambda^2\right) = \underline{\underline{1/3\,\lambda^3}}$

Da eine quadratische Funktion exakt integriert wird, ist der Genauigkeitsgrad der Quadraturformel mind. 2.

b) Genauigkeitgrad Gauss: $2n-1 \stackrel{!}{=} 9 \Rightarrow \underline{\underline{n=5.}}$

c)

h_0 | $R_{0,0}=2$

$h_1 = \frac{h_0}{2}$ | $R_{1,0}=1{,}5$ — $R_{1,1} = \frac{R_{1,0} - 4^{-1} R_{0,0}}{1 - 4^{-1}} = 4/3$

$h_2 = \frac{h_0}{4}$ | $R_{2,0}=1{,}3$ — $R_{2,1} = \frac{R_{2,0} - 4^{-1} R_{1,0}}{1 - 4^{-1}} = 1.2333$ — $R_{2,2} = \frac{R_{2,1} - 4^{-2} R_{1,1}}{1 - 4^{-2}}$

$= \underline{\underline{1.2266}}$

(Skript p. 98)

A3

a) (Skript p. 68)

$$P_2(x) = f_0 l_0(x) + f_1 l_1(x) + f_2 l_2(x)$$

mit Stützstellen $x_0 = 1, \quad x_1 = 1.5, \quad x_2 = 2,$

$f_0 = f(x_0) = 1, \quad f_1 = f(x_1) = \frac{2}{3}, \quad f_2 = f(x_2) = \frac{1}{2}.$

Also: $l_0(x) = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} = + \frac{x - 1.5}{0.5} \cdot \frac{x - 2}{1}$

$l_1(x) = \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_0 - x_2} = - \frac{x - 1}{0.5} \cdot \frac{x - 2}{1}$

$l_2(x) = \frac{x - x_0}{x_2 - x_0} \cdot \frac{x - x_1}{x_2 - x_1} = \frac{x - 1}{1} \cdot \frac{x - 1.5}{0.5}$

$$\Rightarrow P_2(x) = \left(\frac{x - 1.5}{0.5}\right)(x - 2) - \frac{2}{3}\left(\frac{x - 1}{0.5}\right)(x - 2) + \frac{1}{2}(x - 1)\left(\frac{x - 1.5}{0.5}\right)$$

alternativ: Parabel durch die Funktionswerte in x_0, x_1, x_2 legen.

b) $\| f(x) - P_2(x) \| = \left\| \frac{f^{(2+1)}(\xi)}{(2+1)!} (x - 1)(x - 1.5)(x - 2) \right\| \leq \| (x-1)(x-1.5)(x-2) \|.$

$$\| f^{(3)}(\xi) \| = \left\| \left(- \frac{1}{\xi^2} \right)'' \right\| = \left\| \left(2 \frac{1}{\xi^3} \right)' \right\| = \left\| -6 \frac{1}{\xi^4} \right\| \leq 6$$

A4

a) Fehlergleichungen:

$$a \cdot 0 + b \cdot 0 + c - 10^3 = r_1$$
$$a \cdot 1 + b \cdot 1 + c - 3\, 10^3 = r_2$$
$$a \cdot 2 + b \cdot 4 + c - 6 \cdot 10^3 = r_3$$

$$a \cdot 3 + b \cdot 9 + c - 10^4 = r_4$$
$$a \cdot 4 + b \cdot 16 + c - 10^5 = r_5$$

$$\Rightarrow \quad A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \\ 4 & 16 & 1 \end{pmatrix}.$$

b)

$$\begin{pmatrix} 1/\sqrt{2} & \sqrt{2} \\ 0 & 2\sqrt{2} \\ -1/\sqrt{2} & -\sqrt{2} \end{pmatrix}$$

$$A = Q \cdot R \quad \Rightarrow \quad R = Q^T \cdot A = \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} -1 & -2 \\ 0 & -2\sqrt{2} \\ 0 & 0 \end{pmatrix}$$

$$\| Ax - c \| = \| QRx - c \| = \| Rx - Q^T c \| \to \min.$$

$$Q^T c = \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & -1 & 0 \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \sqrt{2} \\ 3 \\ \sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$$

$$\Rightarrow \quad \begin{pmatrix} -1 & -2 \\ 0 & -2\sqrt{2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \overset{!}{=} \begin{pmatrix} 0 \\ -3 \end{pmatrix} \quad \Rightarrow \quad x_2 = 3/2\sqrt{2}$$

$$x_1 = -2x_2 = -3/\sqrt{2}$$

A5:

NM WS 2012

a) (Skript p. 121) $k_1 = f(t + \frac{1}{4}h,\ x_0 + \frac{1}{4}h k_1)$

$$\bar{x} = x_0 + h k_1$$

$$\Rightarrow \quad k_1 = (t + \tfrac{1}{4}h)^2 + 5(x_0 + \tfrac{1}{4}h k_1)$$

$$\Rightarrow \quad k_1 = \frac{(t + \frac{1}{4}h)^2 + 5x}{1 - \cancel{1}5/4\, h} \quad \Rightarrow \quad \underline{\underline{\bar{x} = x_0 + h\frac{(t + \frac{1}{4}h)^2 + 5x_0}{1 - 5/4 h}}}$$

b) (Skript p. 126) $f(t, x) = \lambda x$

$$\Rightarrow \quad k_1 = \lambda x$$

$$k_2 = \lambda(x + h\lambda x)$$

$$k_3 = \lambda\left(x + \frac{h}{3}\lambda x + \frac{h}{3}k_3\right) \Rightarrow k_3 = \frac{\lambda(x + \frac{h}{3}\lambda x)}{1 - \frac{\lambda h}{3}}$$

$$\bar{x} = x + \frac{h}{2}\lambda(x + h\lambda x) + \frac{h}{2}\frac{\lambda(x + \frac{h}{3}\lambda x)}{1 - \frac{\lambda h}{3}}$$

$$= x \underbrace{\left(1 + \frac{\lambda h}{2} + \frac{(\lambda h)^2}{2} + \frac{\lambda h/2}{1 - \frac{\lambda h}{3}}\left(1 + \lambda h/3\right)\right)}_{\text{Stabilitätsfunktion.}}$$

A6

NM WS2012

a)

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x = 0 : dx : 2;
N_x = length(x);
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b)

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q = ((x.*y) >= 0) - ((x.*y) < 0);
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