Sheet 6

Unless stated otherwise we work over an algebraically closed field k.

1. Let $p, q \geq 2$ be coprime integers. Compute the proper transform of the curve $x^p + y^q = 0$ in $\mathbb{A}^2$ under the blowup map $\widetilde{\mathbb{A}^2} = \mathrm{Bl}_0 \mathbb{A}^2 \to \mathbb{A}^2$. This will show that for every $k \geq 1$ there exist curves which are singular after iteratively blowing up k times the singular points.

2. *(Harris Exercise 7.27)* Consider the projection of a quadric hypersurface $Q \subseteq \mathbb{P}^n$ from a point $p \in Q$. Assume that the quadratic form has full rank. Describe the birational isomorphism $Q \dashrightarrow \mathbb{P}^{n-1}$ (sheet 5, exercise **5**) in terms of blowing up and blowing down.

3. *(Harris Exercise 7.28)* Consider the birational equivalence

$$\mathbb{P}^m \times \mathbb{P}^n \dashrightarrow \mathbb{P}^{m+n}, ([z_0, \dots, z_m], [w_0, \dots, w_n]) \mapsto [z_0 w_0, z_1 w_0, \dots, z_m w_0, z_0 w_1, \dots, z_0 w_n] .$$

Describe the graph of this map and describe the map in terms of blowing up and down.

4. *(Standard quadratic transformation I, Hartshorne Exercise I.4.6)* The *standard quadratic transformation* is the rational map $\varphi : \mathbb{P}^2 \dashrightarrow \mathbb{P}^2$, $[a_0, a_1, a_2] \mapsto [a_1 a_2, a_0 a_2, a_0 a_1]$.

a) Show that φ is birational, and is its own inverse.

b) Find open sets $U, V \subseteq \mathbb{P}^2$ such that $\varphi : U \to V$ is an isomorphism.

c) Find the open sets where φ and φ^{-1} are defined, and describe the corresponding morphisms.

5. *(Standard quadratic transformation II)* Show that the standard quadratic transformation φ from the previous exercise extends to an isomorphism from $\mathrm{Bl}_{\{[1,0,0],[0,1,0],[0,0,1]\}} \mathbb{P}^2$ to itself.

Due on Friday, April 17.