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Multiple Choice Questions

You should be able to identify the correct statements, and to justify the answers (either by a counterexample, or a reference to a theorem proved in class).

1. Let L/K be a finite extension of fields. Which of the following assertions are correct:

A. If the characteristic of K is zero, then L/K is normal.

B. If the characteristic of K is zero, then L/K is separable.

C. If L/K is normal, then L/K is a Galois extension.

D. If the characteristic of K is positive, then L/K is normal if and only if it is separable.

2. Let L/K be a finite extension of fields. Which of the following assertions are correct:

A. If $L = K(x)$, where x is a root of a separable polynomial in $K[X]$, then L/K is separable.

B. There exists $x \in L$ such that $L = K(x)$.

C. For any embedding σ of K in an algebraic closed-field $\bar{K}$, there exists $\tau : L \to \bar{K}$ which extends σ.

3. Is it true that if K is a finite field, then any finite extension L/K is a Galois extension? What about any algebraic extension?

4. Let K be a field, $\bar{K}$ an algebraic closure of K and $P \in K[X]$ a non-constant polynomial. Let $L \subset \bar{K}$ denote the splitting field of P in $\bar{K}$. Which of the following assertions are correct:

A. The extension L/K is a normal extension.

B. If $x \in \bar{K}$ is a root of P, then $L = K(x)$.

C. The extension L/K is a Galois extension.

D. If the polynomial P is irreducible, then L/K is a Galois extension.

E. If the characteristic of K is zero, then L/K is a Galois extension.

5. Let K be a field, $\bar{K}$ an algebraic closure of K and $L \subset \bar{K}$ a finite extension of K such that L/K is a Galois extension. Let $K \subset E \subset L$ be an intermediate extension. Which of the following assertions are correct:

A. The extension L/E is a Galois extension.

B. The extension E/K is a normal extension.

C. The extension E/K is a separable extension.

6. Let K be a field, $\bar{K}$ an algebraic closure of K and $L \subset \bar{K}$ a finite extension of K such that L/K is a Galois extension, and let G be its Galois group. Which of the following assertions are correct:

A. For any subgroup H of G, the intermediate extension $E = L^H$ is a normal extension of K.

B. Two subgroups H_1 and H_2 of G are equal if and only if $L^{H_1} = L^{H_2}$.

C. Any subgroup H of G is the Galois group of some extension E/K for some $E \subset L$.

D. Any subgroup H of G is the Galois group of some extension L/E for some $E \subset L$.

7. Let K be a field, $\bar{K}$ an algebraic closure of K and $L \subset \bar{K}$ a finite extension of K such that L/K is a Galois extension, and let G be its Galois group. Let $x \in L$ be given and $\sigma_0 \in G$ a non-trivial element. Which of the following assertions are correct:

A. If $\sigma_0(x) = x$, then $x \in K$.

B. If G is cyclic and $\sigma_0(x) = x$, then $x \in K$.

C. The element

$$\sum_{\sigma \in G} \sigma(x)^2$$

belongs to K.

D. If the set of all $\sigma(x)$, for σ ranging over G, contains at most two elements, then $[K(x) : K] \leq 2$.

8. Let K be a field, $\bar{K}$ an algebraic closure of K and $L \subset \bar{K}$ a finite extension of K of degree 2. Which of the following assertions are correct:

A. The extension L/K is separable.

B. The extension L/K is normal.

C. If the characteristic of K is zero, then there exists $y \in L$ such that $L = K(y)$ and $y^2 \in K^\times$.