

Brownian Motion and Stochastic Calculus

Sketch of Solution Sheet 12

- For $m \geq 1$ large enough so that $\frac{1}{m} < d(x, U^c)$, we define $T_m := \inf \{s \geq 0 ; d(X_s^x, U^c) \leq \frac{1}{m}\}$ and construct $u_m \in C_c^2(\mathbb{R}^d, \mathbb{R})$ such that $u = u_m$ on $\{z \in U ; d(z, U^c) \geq \frac{1}{m}\}$. We apply Itô's formula to $u_m(X_t^x) \exp \left(\int_0^t c(X_s^x) ds \right)$, take then the expectation and use that the local martingale is a true martingale as b is locally bounded and $u \in C_c^2$ (you can check it for example via Burkholder-Davis-Gundy) to obtain that

$$\begin{aligned} & E \left[u_m(X_{t \wedge T_m^x}^x) \exp \left(\int_0^{t \wedge T_m^x} c(X_s^x) ds \right) \right] - u_m(x) \\ &= E \left[\int_0^{t \wedge T_m^x} (Lu_m(X_s^x) + c(X_s^x)u_m(X_s^x)) \exp \left(\int_0^s c(X_r^x) dr \right) ds \right]. \end{aligned}$$

Now, as $u_m = u$ on $\{z \in U ; d(z, U^c) \geq \frac{1}{m}\}$, by definition of T_m^x and as u is the solution of the boundary problem, we obtain that

$$u(x) = E \left[u(X_{t \wedge T_m^x}^x) \exp \left(\int_0^{t \wedge T_m^x} c(X_s^x) ds \right) \right] + E \left[\int_0^{t \wedge T_m^x} f(X_s^x) \exp \left(\int_0^s c(X_r^x) dr \right) ds \right].$$

Since $T_m^x \uparrow T_U^x < \infty$, we can let $t \rightarrow \infty$ and then $m \rightarrow \infty$ to conclude, by the dominated convergence theorem, that

$$u(x) = E \left[g(X_{T_U^x}^x) \exp \left(\int_0^{T_U^x} c(X_s^x) ds \right) \right] + E \left[\int_0^{T_U^x} f(X_s^x) \exp \left(\int_0^s c(X_r^x) dr \right) ds \right].$$

- a) Note that

$$\lim_{s \rightarrow t} P[|X_t - X_s| > \varepsilon] = \lim_{s \rightarrow t} P[|X_{|t-s|}| > \varepsilon] = \lim_{h \downarrow 0} P[|X_h| > \varepsilon] = 0,$$

because X is RC a.s.

- b) $f_0(u) = 1$ is clear. Using independence and stationarity of the increments as well as $X_0 = 0$ P -a.s., we have for any $s, t \geq 0$

$$\begin{aligned} f_{s+t}(u) &= E[\exp(iu^{\text{tr}} X_{s+t})] = E[\exp(iu^{\text{tr}}(X_{s+t} - X_s)) \exp(iu^{\text{tr}} X_s)] \\ &= E[\exp(iu^{\text{tr}}(X_{s+t} - X_s))] E[\exp(iu^{\text{tr}} X_s)] = E[\exp(iu^{\text{tr}} X_t)] E[\exp(iu^{\text{tr}} X_s)] \\ &= f_t(u) f_s(u). \end{aligned}$$

- c) Let $m, n \in \mathbb{N}$. Using the property $f_{s+t}(u) = f_s(u)f_t(u)$ inductively, it follows that

$$(f_{m/n}(u))^n = f_m(u) = f_1(u)^m.$$

Hence, $f_{m/n}(u) = f_1(u)^{m/n}$.

- d) Right-continuity of $t \mapsto f_t(u)$ follows immediately from right-continuity of X and the bounded convergence theorem. Moreover, the function $t \mapsto f_1(u)^t$ is continuous and by part c), $f_t(u) = f_1(u)^t$ for all $t \in \mathbb{Q}_+$. It follows that $f_t(u) = f_1(u)^t$ for all $t \geq 0$. Now, assume $f_t(u) = 0$ for some $t > 0$ and $u \in \mathbb{R}^d$. Then it follows that $f_{t/n}(u) = f_t(u)^{1/n} = 0$ for all $n \in \mathbb{N}$. Taking $n \rightarrow \infty$, we obtain a contradiction to the right-continuity.

- e) The integrability implies that the characteristic function is differentiable in u , and then $X_t \in L^1$ for all t . Moreover,

$$i E[X_t] = \partial_u f_t(u)|_{u=0} = t\psi'(u)e^{t\psi(u)}|_{u=0} = t\psi'(0) = i t E[X_1].$$

3. a) We first show the independence of the increments. Let $n \in \mathbb{N}$, $0 \leq t_0 < t_1 \dots < t_n$ and $(f_i)_{i=1}^n$ Borel measurable functions. We need to show that

$$E \left[\prod_{i=1}^n f_i(X_{t_i} - X_{t_{i-1}}) \right] = \prod_{i=1}^n E[f_i(X_{t_i} - X_{t_{i-1}})].$$

Let \mathcal{G} the σ -field generated by $(N_t)_{t \geq 0}$. Then, by independence of the (Y_j) to N , we obtain that

$$\begin{aligned} E \left[\prod_{i=1}^n f_i(X_{t_i} - X_{t_{i-1}}) \right] &= E \left[\prod_{i=1}^n f_i \left(\sum_{j=N_{t_{i-1}}+1}^{N_{t_i}} Y_j \right) \right] \\ &= E \left[E \left[\prod_{i=1}^n f_i \left(\sum_{j=N_{t_{i-1}}+1}^{N_{t_i}} Y_j \right) \mid \mathcal{G} \right] \right] \\ &= E \left[E \left[\prod_{i=1}^n f_i \left(\sum_{j=n_{i-1}+1}^{n_i} Y_j \right) \right] \middle|_{n_i=N_{t_i}, n_{i-1}=N_{t_{i-1}}} \right]. \end{aligned}$$

Now, since the (Y_j) are i.i.d. we obtain that

$$\begin{aligned} E \left[E \left[\prod_{i=1}^n f_i \left(\sum_{j=n_{i-1}+1}^{n_i} Y_j \right) \right] \middle|_{n_i=N_{t_i}, n_{i-1}=N_{t_{i-1}}} \right] &= E \left[\prod_{i=1}^n E \left[f_i \left(\sum_{j=1}^{n_i-n_{i-1}} Y_j \right) \right] \middle|_{n_i=N_{t_i}, n_{i-1}=N_{t_{i-1}}} \right] \\ &= E \left[\prod_{i=1}^n E \left[f_i \left(\sum_{j=1}^{m_i} Y_j \right) \right] \middle|_{m_i=N_{t_i}-N_{t_{i-1}}} \right]. \end{aligned}$$

Siehe nächstes Blatt!

As (N_t) has independent increments we obtain that

$$\begin{aligned} E\left[\prod_{i=1}^n E\left[f_i\left(\sum_{j=1}^{m_i} Y_j\right)\right] = \prod_{i=1}^n E\left[E\left[f_i\left(\sum_{j=n_{i-1}+1}^{n_i} Y_j\right)\right]\Big|_{n_i=N_{t_i}, n_{i-1}=N_{t_{i-1}}}\right] \\ = \prod_{i=1}^n E\left[f_i(X_{t_i} - X_{t_{i-1}})\right]. \end{aligned}$$

Now, we show that X has stationary increments. As X has independent increments, it's enough to show that for $s < t$ and f Borel, we have

$$E[f(X_t - X_s)] = E[f(X_{t-s})].$$

With the same arguments we used for showing the independence of the increments of X , using that (N_t) has stationary increments, we obtain that

$$\begin{aligned} E[f(X_t - X_s)] &= E\left[f\left(\sum_{j=N_s+1}^{N_t} Y_j\right)\right] = E\left[E\left[f\left(\sum_{j=N_s+1}^{N_t} Y_j\right)\Big| \mathcal{G}\right]\right] \\ &= E\left[E\left[f\left(\sum_{j=n_s+1}^{n_t} Y_j\right)\right]\Big|_{n_t=N_t, n_s=N_s}\right] \\ &= E\left[E\left[f\left(\sum_{j=1}^{n_t-n_s} Y_j\right)\right]\Big|_{n_t=N_t, n_s=N_s}\right] \\ &= E\left[E\left[f\left(\sum_{j=1}^{m_{t,s}} Y_j\right)\right]\Big|_{m_{t,s}=N_t-N_s}\right] \\ &= E\left[E\left[f\left(\sum_{j=1}^{m_{t,s}} Y_j\right)\right]\Big|_{m_{t,s}=N_{t-s}}\right] \\ &= E\left[f\left(\sum_{j=1}^{N_{t-s}} Y_j\right)\right] \\ &= E[f(X_{t-s})]. \end{aligned}$$

We conclude that X is a Lévy process. We proceed with calculating its triplet.

Bitte wenden!

For $u \in \mathbb{R}^d$,

$$\begin{aligned}
E[e^{iu^{\text{tr}}X_t}] &= E\left[\sum_{k \geq 0} 1_{N_t=k} \prod_{j=1}^k e^{iu^{\text{tr}}Y_j}\right] \\
&= \sum_{k \geq 0} P[N_t = k] E[e^{iu^{\text{tr}}Y_1}]^k \\
&= \sum_{k \geq 0} e^{-\lambda t} \frac{(\lambda t)^k}{k!} E[e^{iu^{\text{tr}}Y_1}]^k \\
&= e^{-\lambda t} \exp\left(\lambda t E[e^{iu^{\text{tr}}Y_1}]\right) \\
&= \exp\left(\lambda t \{E[e^{iu^{\text{tr}}Y_1}] - 1\}\right).
\end{aligned}$$

If F is the distribution of Y_1 and $\nu := \lambda F$, we get

$$E[e^{iu^{\text{tr}}X_t}] = \exp\left(t \int_{\mathbb{R}} (e^{iu^{\text{tr}}x} - 1) d(\lambda F(x))\right).$$

Therefore, the triplet is $(b, 0, \nu)$, where $b = \int_{\{x: |x| \leq 1\}} x d\nu$.

- b)** The characteristic function is $f_1(u) = \phi_{X_1}(u) = (e^{iu} - 1)(iu)^{-1}$, which has a zero at $u = 2\pi$. This would contradict Ex 12-2 d), hence there is no such Lévy process.
- c)** Let $n \in \mathbb{N}$ and $0 = t_0 < t_1 < \dots < t_n$. We need to show that $(X_{t_1}, \dots, X_{t_n})$ is independent of $(Y_{t_1}, \dots, Y_{t_n})$. In the first step, we show that

$$\begin{aligned}
&E\left[\exp\left(i \sum_{k=1}^n u_k^{\text{tr}}(X_{t_k} - X_{t_{k-1}})\right) \exp\left(i \sum_{k=1}^n v_k^{\text{tr}}(Y_{t_k} - Y_{t_{k-1}})\right)\right] \\
&= E\left[\exp\left(i \sum_{k=1}^n u_k^{\text{tr}}(X_{t_k} - X_{t_{k-1}})\right)\right] E\left[\exp\left(i \sum_{k=1}^n v_k^{\text{tr}}(Y_{t_k} - Y_{t_{k-1}})\right)\right].
\end{aligned}$$

We use an induction argument. For $n = 1$, this is the assumption. assume that it holds true for $n - 1$. We obtain, as the increments are independent of the past, that

$$\begin{aligned}
&E\left[\exp\left(i \sum_{k=1}^n u_k^{\text{tr}}(X_{t_k} - X_{t_{k-1}})\right) \exp\left(i \sum_{k=1}^n v_k^{\text{tr}}(Y_{t_k} - Y_{t_{k-1}})\right)\right] \\
&= E\left[\exp\left(i \sum_{k=1}^{n-1} u_k^{\text{tr}}(X_{t_k} - X_{t_{k-1}})\right) \exp\left(i \sum_{k=1}^{n-1} v_k^{\text{tr}}(Y_{t_k} - Y_{t_{k-1}})\right)\right] \\
&\quad \cdot E\left[\exp\left(i u_n^{\text{tr}}(X_{t_n} - X_{t_{n-1}})\right) \exp\left(i v_n^{\text{tr}}(Y_{t_n} - Y_{t_{n-1}})\right)\right].
\end{aligned}$$

Siehe nächstes Blatt!

Now, using the induction hypothesis, we obtain that

$$\begin{aligned}
&= E \left[\exp \left(i \sum_{k=1}^{n-1} u_k^{\text{tr}} (X_{t_k} - X_{t_{k-1}}) \right) \exp \left(i \sum_{k=1}^{n-1} v_k^{\text{tr}} (Y_{t_k} - Y_{t_{k-1}}) \right) \right] \\
&\quad \cdot E \left[\exp \left(i u_n^{\text{tr}} (X_{t_n} - X_{t_{n-1}}) \right) \exp \left(i v_n^{\text{tr}} (Y_{t_n} - Y_{t_{n-1}}) \right) \right]. \\
&= E \left[\exp \left(i \sum_{k=1}^{n-1} u_k^{\text{tr}} (X_{t_k} - X_{t_{k-1}}) \right) \right] E \left[\exp \left(i \sum_{k=1}^{n-1} v_k^{\text{tr}} (Y_{t_k} - Y_{t_{k-1}}) \right) \right] \\
&\quad \cdot E \left[\exp \left(i u_n^{\text{tr}} (X_{t_n} - X_{t_{n-1}}) \right) \exp \left(i v_n^{\text{tr}} (Y_{t_n} - Y_{t_{n-1}}) \right) \right].
\end{aligned}$$

Using again the independence of increments of the past of (\mathcal{F}_t) , we obtain that

$$\begin{aligned}
&E \left[\exp \left(i \sum_{k=1}^{n-1} u_k^{\text{tr}} (X_{t_k} - X_{t_{k-1}}) \right) \right] E \left[\exp \left(i \sum_{k=1}^{n-1} v_k^{\text{tr}} (Y_{t_k} - Y_{t_{k-1}}) \right) \right] \\
&\quad \cdot E \left[\exp \left(i u_n^{\text{tr}} (X_{t_n} - X_{t_{n-1}}) \right) \exp \left(i v_n^{\text{tr}} (Y_{t_n} - Y_{t_{n-1}}) \right) \right] \\
&= E \left[\exp \left(i \sum_{k=1}^{n-1} u_k^{\text{tr}} (X_{t_k} - X_{t_{k-1}}) \right) \right] E \left[\exp \left(i \sum_{k=1}^{n-1} v_k^{\text{tr}} (Y_{t_k} - Y_{t_{k-1}}) \right) \right] \\
&\quad \cdot \frac{E \left[\exp \left(i u_n^{\text{tr}} X_{t_n} \right) \exp \left(i v_n^{\text{tr}} Y_{t_n} \right) \right]}{E \left[\exp \left(i u_n^{\text{tr}} X_{t_{n-1}} \right) \exp \left(i v_n^{\text{tr}} Y_{t_{n-1}} \right) \right]}.
\end{aligned}$$

Now, using the assumption that $E[e^{i u^{\text{tr}} X_t} e^{i v^{\text{tr}} Y_t}] = E[e^{i u^{\text{tr}} X_t}] E[e^{i v^{\text{tr}} Y_t}]$ for all $u, v \in \mathbb{R}^d$ and $t \geq 0$, we obtain that

$$\begin{aligned}
&E \left[\exp \left(i \sum_{k=1}^{n-1} u_k^{\text{tr}} (X_{t_k} - X_{t_{k-1}}) \right) \right] E \left[\exp \left(i \sum_{k=1}^{n-1} v_k^{\text{tr}} (Y_{t_k} - Y_{t_{k-1}}) \right) \right] \\
&\quad \cdot \frac{E \left[\exp \left(i u_n^{\text{tr}} X_{t_n} \right) \exp \left(i v_n^{\text{tr}} Y_{t_n} \right) \right]}{E \left[\exp \left(i u_n^{\text{tr}} X_{t_{n-1}} \right) \exp \left(i v_n^{\text{tr}} Y_{t_{n-1}} \right) \right]} \\
&= E \left[\exp \left(i \sum_{k=1}^{n-1} u_k^{\text{tr}} (X_{t_k} - X_{t_{k-1}}) \right) \right] E \left[\exp \left(i \sum_{k=1}^{n-1} v_k^{\text{tr}} (Y_{t_k} - Y_{t_{k-1}}) \right) \right] \\
&\quad \cdot \frac{E \left[\exp \left(i u_n^{\text{tr}} X_{t_n} \right) \right] E \left[\exp \left(i v_n^{\text{tr}} Y_{t_n} \right) \right]}{E \left[\exp \left(i u_n^{\text{tr}} X_{t_{n-1}} \right) \right] E \left[\exp \left(i v_n^{\text{tr}} Y_{t_{n-1}} \right) \right]}
\end{aligned}$$

Bitte wenden!

using twice the independence of increments of the past of (\mathcal{F}_t) yields that

$$\begin{aligned}
& E \left[\exp \left(i \sum_{k=1}^{n-1} u_k^{\text{tr}} (X_{t_k} - X_{t_{k-1}}) \right) \right] E \left[\exp \left(i \sum_{k=1}^{n-1} v_k^{\text{tr}} (Y_{t_k} - Y_{t_{k-1}}) \right) \right] \\
& \cdot \frac{E \left[\exp \left(i u_n^{\text{tr}} X_{t_n} \right) \right] E \left[\exp \left(i v_n^{\text{tr}} Y_{t_n} \right) \right]}{E \left[\exp \left(i u_n^{\text{tr}} X_{t_{n-1}} \right) \right] E \left[\exp \left(i v_n^{\text{tr}} Y_{t_{n-1}} \right) \right]} \\
& = E \left[\exp \left(i \sum_{k=1}^{n-1} u_k^{\text{tr}} (X_{t_k} - X_{t_{k-1}}) \right) \right] E \left[\exp \left(i \sum_{k=1}^{n-1} v_k^{\text{tr}} (Y_{t_k} - Y_{t_{k-1}}) \right) \right] \\
& \cdot E \left[\exp \left(i u_n^{\text{tr}} (X_{t_n} - X_{t_{n-1}}) \right) \right] E \left[\exp \left(i v_n^{\text{tr}} (Y_{t_n} - Y_{t_{n-1}}) \right) \right] \\
& = E \left[\exp \left(i \sum_{k=1}^n u_k^{\text{tr}} (X_{t_k} - X_{t_{k-1}}) \right) \right] E \left[\exp \left(i \sum_{k=1}^n v_k^{\text{tr}} (Y_{t_k} - Y_{t_{k-1}}) \right) \right].
\end{aligned}$$

So we have proved the claim. Next, since characteristic functions determine the law of random vectors, we conclude that $\bar{X} := (X_{t_1} - X_{t_0}, \dots, X_{t_n} - X_{t_{n-1}})$ and $\bar{Y} := (Y_{t_1} - X_{t_0}, \dots, Y_{t_n} - Y_{t_{n-1}})$ are independent. As $X_{t_0} = Y_{t_0} = 0$ we can find a linear (and hence continuous) function f such that $f(\bar{X}) = (X_{t_1}, X_{t_2}, \dots, X_{t_n})$ and $f(\bar{Y}) = (Y_{t_1}, Y_{t_2}, \dots, Y_{t_n})$. Thus we conclude that $(X_{t_1}, \dots, X_{t_n})$ and $(Y_{t_1}, \dots, Y_{t_n})$ are independent, which was to show.

4. Matlab Files

(a) Applying the definition of expectation we have

$$\begin{aligned}
\mathbb{E}[e^{N_1}] &= \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} e^k \\
&= e^{-\lambda} \sum_{k=0}^{\infty} \frac{(\lambda e)^k}{k!} \\
&= e^{\lambda(e-1)}.
\end{aligned}$$

Plugging in $\lambda = 2$ we get $\mathbb{E}[e^{N_1}] = 31.08$.

(d) For the compound Poisson process we have to condition on N_1 , i.e.,

$$\begin{aligned}
 \mathbb{E}[e^{X_1}] &= \mathbb{E}[\mathbb{E}[e^{X_1}|N_1]] \\
 &= \sum_{k=0}^{\infty} \mathbb{E}[e^{X_1}|N_t] \mathbb{P}[N_1 = k] \\
 &= \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!} \mathbb{E}[\prod_{j=1}^k e^{Z_j}] \\
 &= e^{-\lambda} \sum_{k=0}^{\infty} \frac{(\lambda\beta)^k}{k!} \\
 &= e^{\lambda(\beta-1)},
 \end{aligned}$$

where $\beta = \mathbb{E}[e^{Z_1}] = 0.5(e + e^{-1})$. Plugging in $\lambda = 2$ we get $\mathbb{E}[e^{X_1}] = 2.96$.

```

1 function [simulatedvalue , theoreticalvalue ]= bmsc124a
2 % In this exercise we simulated 10 paths of Poisson
   process with
3 % intensity lambda=2
4 tic
5 %% parameter input
6 % horizon
7 T=1;
8 % sample size
9 Nplot=10;
10 Nsimu=10^5;
11 % grid points
12 M=10^3;
13 % intensity
14 lambda=2;
15
16 % theoretical value
17 theoreticalvalue= exp(lambda*(exp(1)-1));
18 %% Simulation
19 % method 1
20 figure(1)
21 % number of total jumps at T
22 NT= poissrnd(lambda*T,1,Nplot);
23 temp1= zeros(max(NT),Nplot);
24 for i =1:Nplot
25     %condition on N_T, the jump times are ordered
       %uniformlly distributed
26     temp1(1:NT(i),i)= sort(T*rand(1,NT(i)));
27     stairs([0,temp1(1:NT(i),i)],(0:NT(i))');

```

Bitte wenden!

```

28     hold on
29 end
30 hold off
31
32 % method 2
33 PP = [zeros(1,Nplot);cumsum(poissrnd(lambda*T/M,M,Nplot)
    )];
34 % the process  $X^c$ 
35 timegrid= 0:T/M:T;
36 Xc=PP;
37
38 figure(2)
39 %show the first 10 sample paths
40 plot(timegrid ,Xc(:, :))
41
42
43 %compute simulated value
44 poissonrv=poissrnd(lambda*T,1,Nsimu);
45 simulatedvalue= mean(exp(poissonrv));
46 toc

1 function [simulatedvalue, theoreticalvalue] = bmsc124b
2 % In this exercise we simulated 10 paths of compound
   Poisson process
3 %  $X^c(t) = \sum_{i=1}^N Z_i$  with intensity lambda=2 and
   Z_i = 1/-1 with
4 % prob=0.5
5 tic
6 %% parameter input
7 % horizon
8 T=1;
9 % sample size
10 Nplot=10;
11 Nsimu=10^5;
12 % grid points
13 M=10^3;
14 % intensity
15 lambda=2;
16 % expectation of  $e^Z$ 
17 beta = 0.5*(exp(1)+exp(-1));
18
19 % theoretical value
20 theoreticalvalue= exp(lambda*T*(beta -1));

```

Siehe nächstes Blatt!

```

21 %% Simulation
22 % compound poisson process
23 figure(1)
24 % method 1
25 % number of total jumps at T
26 NT= poissrnd(lambda*T,1,Nplot);
27 temp1= zeros(max(NT),Nplot);
28 for i =1:Nplot
29     %condition on N_T, the jump times are ordered
        %uniformlly distributed
30     temp1(1:NT(i),i)= sort(T*rand(1,NT(i)));
31     % simulate Z
32     stairs([0,temp1(1:NT(i),i)],cumsum([0,2*unidrnd(2,1,
            NT(i))-3])')
33     hold on;
34 end
35 hold off;
36
37
38 % method 2
39 figure(2)
40 CPP = [zeros(1,Nplot);cumsum(poissrnd(lambda*T/M,M,Nplot
            ).*(2*unidrnd(2,M,Nplot)-3))];
41 % the process X^(a)
42 timegrid= 0:T/M:T;
43 Xc=CPP;
44
45 %show the first 10 sample paths
46 plot(timegrid,Xc(:,1:10))
47
48 %compute simulated value
49 % the poisson process N
50 poissonrv=poissrnd(lambda*T,1,Nsimu);
51 maxN=max(poissonrv);
52 % the jump distribution Z
53 bernoullirv= 2*unidrnd(2,maxN,Nsimu)-3;
54 % place holder
55 temp= zeros(1,Nsimu);
56 for i=1:maxN
57     % we only consider those numbers for which the
        % poisson process is > 0
58     index= (poissonrv>0);

```

Bitte wenden!

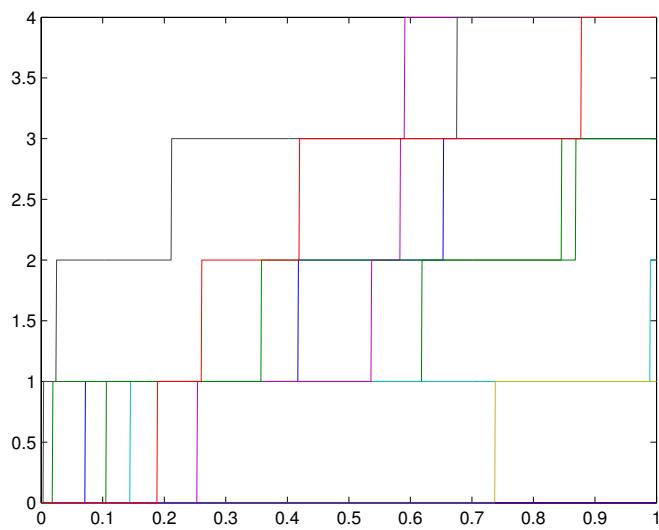


Abbildung 1: 10 sample path a Poisson process with intensity $\lambda = 2$

```

59      temp(index)=temp(index)+bernoullirv(i,index);
60      poissonrv(index)=poissonrv(index)-1;
61 end
62 simulatedvalue= mean(exp(temp));
63 toc

```

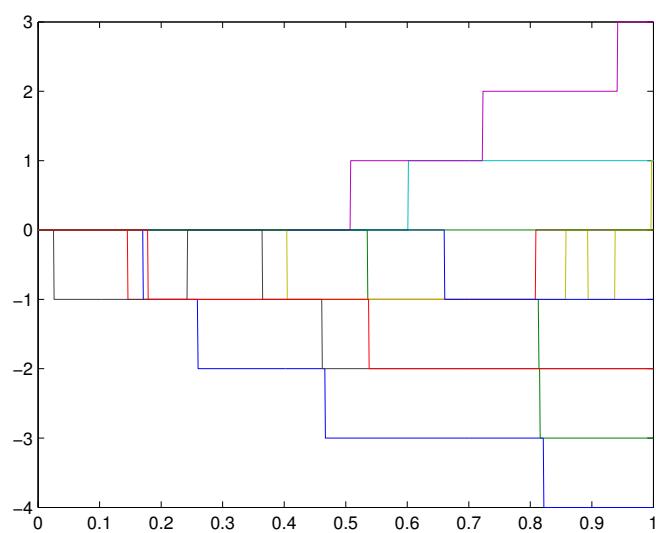


Abbildung 2: 10 sample path a compound Poisson process with intensity $\lambda = 2$