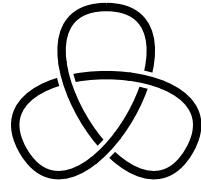
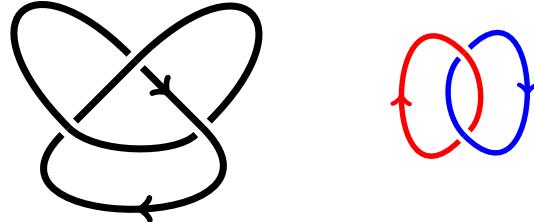


Examplesheet 4: The Jones Polynomial

1. Prove that the Kauffman bracket is invariant under Reidemeister move three.
2. Calculate the bracket polynomial of the usual projection of the trivial link with n components.
3. Determine the bracket polynomial for the following projection of the trefoil.

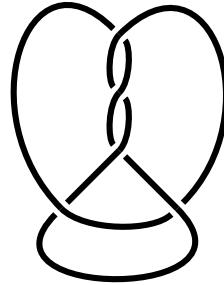


4. (a) Show the writhe of a link projection is invariant under Reidemeister moves two and three.
 (b) Prove the writhe of a link projection has the same parity as the number of crossings c in this projection.
5. Compute the X -polynomial of the following two oriented links. What happens to $X(L)$ if we change the orientation on one or on both components of the links?



6. Determine the Jones polynomial of the trefoil – you can use the result of question 3 – what happens if you reverse your chosen orientation?
7. Let c be the actual number of crossings in a given diagram.
 - (a) Prove that $-c \leq \sum s \leq c$.

- (b) Prove that $\sum s$ has the same parity as c .
8. Show that although the Jones polynomial is defined for oriented links, it is actually an invariant for knots (i.e. the orientation is not needed).
 9. Prove the Kauffman bracket is well-defined, i.e. show that the explicit state-sum formula equals the Kauffman bracket.
 10. Compute the Jones polynomial of the figure-eight knot in two ways: first do it by the Kauffman bracket definition and then using the Jones skein relation - you should get the same result and remember the figure-eight knot is amphichiral.
 11. (a) Determine the Jones polynomial of the positive Hopf link (the one with linking number +1) using the Jones skein relation and show that it is not equivalent to its mirror image.
 (b) Calculate the Jones polynomial of the right-handed trefoil using the Jones skein relation and prove (finally) that it is not equivalent to its mirror image.
 12. (a) Prove that $V(K_1 \# K_2) = V(K_1) \cdot V(K_2)$. (Hint: first give a formula for the Kauffman bracket of a connected sum).
 (b) Prove that $V(K_1 \sqcup K_2) = (-t^{\frac{1}{2}} - t^{-\frac{1}{2}})V(K_1) \cdot V(K_2)$, where \sqcup denotes the disjoint union.
 13. Use the axioms to calculate the Jones polynomial of the knot 5_2 shown below – you may use any results you have obtained so far – is this knot amphichiral?



The result should be $V(5_2) = -t^{-6} + t^{-5} - t^{-4} + 2t^{-3} - t^{-2} + t^{-1}$

14. **(Unsolved Question)** Does the Jones polynomial distinguish every knot from the unknot? That is to say, is there a nontrivial knot with Jones polynomial 1?

Due Date: 14.04.2015