Classification of closed surfaces: *Any closed connected surface S is equivalent to exactly one of the surfaces M_g ($g = 0, 1, 2, \ldots$; a sphere with g handles) or N_h ($h = 1, 2, 3, \ldots$; a sphere with h crosscaps). The surfaces can be identified by their orientability and Euler characteristic: the M_g are orientable with $\chi(M_g) = 2 - 2g$, whereas the N_h are non-orientable with $\chi(N_h) = 2 - h$.*

Definition: The **genus** g of a closed surface S is defined by $g(S) = 1 - \frac{1}{2}\chi(S)$ for an orientable surface and by $g(S) = 2 - \chi(S)$ for a non-orientable one.

Classification of closed surfaces with boundary: *Any closed connected surface S with $n \geq 0$ boundary components is equivalent to exactly one of the surfaces M_g^n ($g = 0, 1, 2, \ldots$; an n-punctured sphere with g handles) or N_h^n ($h = 1, 2, 3, \ldots$; an n-punctured sphere with h crosscaps). The surfaces can be identified by their number of boundary components, orientability and Euler characteristic: the M_g^n are orientable with $\chi(M_g^n) = 2 - 2g - n$, whereas the N_h^n are non-orientable with $\chi(N_h^n) = 2 - h - n$.*

For the proofs and further details see e.g. Robert's Notes, section 6.8.