



Graph von  $f(x, y) = \left\{ \begin{array}{l} \frac{x \cdot y \cdot (x^2 - y^2)}{x^2 + y^2} \\ 0 \end{array} \right.$

$x = y$   
 $= 0$

$$\begin{aligned}
 (f_x)_y(0,0) &= \lim_{h \rightarrow 0} \frac{f_x(0,h) - f_x(0,0)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\lim_{k \rightarrow 0} \frac{f(k,h) - f(0,h)}{k} - \lim_{k \rightarrow 0} \frac{f(k,0) - f(0,0)}{k}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\lim_{k \rightarrow 0} \frac{k \cdot h \cdot (k^2 - h^2) - 0}{k(k^2 + h^2)} - 0}{h} \\
 &= \lim_{h \rightarrow 0} \frac{-h}{h} = -1
 \end{aligned}$$

$$\begin{aligned}
 (f_y)_x(0,0) &= \lim_{h \rightarrow 0} \frac{f_y(h,0) - f_y(0,0)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\lim_{k \rightarrow 0} \frac{f(h,k) - f(h,0)}{k} - \lim_{k \rightarrow 0} \frac{f(0,k) - f(0,0)}{k}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\lim_{k \rightarrow 0} \frac{h \cdot k \cdot (h^2 - k^2) - 0}{k(h^2 + k^2)} - 0}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h}{h} = 1
 \end{aligned}$$

Wobei  $f(x,y) = \frac{x \cdot y \cdot (x^2 - y^2)}{(x^2 + y^2)}$

und  $f(0,0) = 0$