Algebraic Geometry
D-MATH, FS 2016

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Exam preparation

1. Consider the variety $\mathbb{C}^n$.
   (a) Define a subvariety X in $\mathbb{C}^n$.
   (b) Can X be isomorphic to $\mathbb{P}^1$?
   (c) Can X be isomorphic to $\mathbb{P}^1 \setminus \{p\}$ (for some $p \in \mathbb{P}^1$)?
   (d) Can X be isomorphic to $\mathbb{P}^1 \setminus \{p, q\}$ (for some $p, q \in \mathbb{P}^1$)?
2. Let L_1, L_2 be two lines in $\mathbb{P}^3$ and $p \in \mathbb{P}^3$ a point. Does there always exist a line L in $\mathbb{P}^3$ which meets L_1, L_2, p?
3. (a) Let I be a proper ideal in $\mathbb{C}[x, y, z]$. Must there be a common zero in $\mathbb{C}^3$ of all polynomials in I?
   (b) Let J be an ideal in $\mathbb{C}[x, y]$ with $(0, 0)$ the only common zero in $\mathbb{C}^2$. Prove that $(x, y)^k$ lies in J for some $k \geq 1$.
4. (a) Let $f : \mathbb{C}^1 \to \mathbb{C}^1$ be a regular map. What are the possibilities for its image $\mathrm{Im}(f)$.
   (b) Let $g : \mathbb{C}^2 \to \mathbb{C}^2$ be a regular map. Is it possible that $\mathrm{Im}(g) = V(xy) \subset \mathbb{C}^2$?
5. What is the ring of algebraic functions on $\mathbb{C}^2 \setminus \{(0, 0)\}$? Is $\mathbb{C}^2 \setminus \{(0, 0)\}$ isomorphic to an affine algebraic variety?
6. Describe the Plucker embedding of $\mathrm{Gr}(2, 4)$ in $\mathbb{P}^5$.
7. Is there a nonconstant regular map $\mathbb{P}^2 \setminus \{p\} \to \mathbb{C}^1$?
8. Can you write the equations of any nonsingular curve in any projective space which is not rational, i.e. birational to $\mathbb{P}^1$? With proof.
9. Consider the scheme $\mathrm{Spec}(\mathbb{Z})$. Find all Zariski open sets. Find the ring of functions on each one.
10. Describe geometrically all maps from $\mathrm{Spec}(\mathbb{C}[z]/(z^2))$ to $\mathrm{Spec}(\mathbb{C}[x, y])$.