Algebraic Geometry

D-MATH, FS 2016

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Exercise Sheet 0

1. Sketch the following subsets of $\mathbb{R}^2$ defined by polynomial equations in the coordinates x, y.

(a) $V(x^2 - x)$

(b) $V(x^2 + 4y^2 - 4)$

(c) $V(x^2 + 4y^2 + 4)$

(d) $V(xy - 1)$

(e) $V(xy)$

(f) $V(y^2 - x^3)$

(g) $V(y^2 - x^2(x + 1))$

(h) $V(y^2 - x^2(x - 1))$

(i) $V(xy - 1, 2y + 2x - 5)$

2. Let $n \geq 1$ and $I, J, I_\alpha \subset \mathbb{C}[x_1, \dots, x_n]$ be ideals ($\alpha \in A$).

(a) Show that $V(I \cdot J) = V(I \cap J) = V(I) \cup V(J)$, where $I \cdot J = (rs : r \in I, s \in J)$ is the ideal generated by products of elements of I and J.

(b) Show that $V(\sum_{\alpha \in A} I_\alpha) = \bigcap_{\alpha \in A} V(I_\alpha)$.

(c) Conclude, that the sets $V(I) \subset \mathbb{C}^n$ (for $I \subset \mathbb{C}[x_1, \dots, x_n]$) form the closed sets of a topology.

(d) Give an example of n and a family I_α as above, such that

$$V(\bigcap_{\alpha \in A} I_\alpha) \neq \bigcup_{\alpha \in A} V(I_\alpha).$$

To be discussed in the exercise class.