Algebraic Geometry

D-MATH, FS 2016

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Exercise Sheet 1

1. Give a careful proof that the ring of functions on an affine, algebraic variety $V \subset \mathbb{C}^n$ is given by $\mathbb{C}[x_1, \ldots, x_n]/I(V)$.

2. For each of the following sets X, find the ideal $I(X)$ of functions vanishing on X and conclude that X is algebraic. Compute their rings of regular functions and show that they are pairwise non-isomorphic as $\mathbb{C}$-algebras.

(a) $X = \{p\} \subset \mathbb{C}^n$ for $p \in \mathbb{C}^n$

(b) $X = \{(t, t^2, t^3) : t \in \mathbb{C}\} \subset \mathbb{C}^3$ (the twisted cubic curve)

(c) $X = \{(t^2, t^3) : t \in \mathbb{C}\} \subset \mathbb{C}^2$ (a cuspidal curve)

3. Show that the ring of algebraic functions on $\mathbb{CP}^n$ are the constant functions $\mathbb{C}$.

4.* Show that the set $\Gamma = \{(z, e^z) : z \in \mathbb{C}\} \subset \mathbb{C}^2$ is not algebraic and determine its closure in the Zariski topology.

Due March 4.