Algebraic Geometry

D-MATH, FS 2016

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Exercise Sheet 5

1. Find the equations defining the image of the following algebraic map.

$$f : \mathbb{P}^1 \to \mathbb{P}^2$$
$$[x, y] \longmapsto [x^3, x^2 y, y^3]$$

2. Let $f = \nu_n : \mathbb{P}^1 \to \mathbb{P}^n, [x, y] \mapsto [x^n, x^{n-1}y, \ldots, y^n]$ be the Veronese map. Let $p_1, \ldots, p_{n+1}$ be *distinct* points in $\mathbb{P}^1$. Prove that $f(p_1), \ldots, f(p_{n+1})$ are in general linear position in $\mathbb{P}^n$.

3. Let $X \subset \mathbb{P}^n$ be a projective variety which is not a finite collection of points. Let $G_d \in \mathbb{C}[x_0, \ldots, x_n]$ be a homogeneous polynomial of degree $d > 0$. Prove that the zero locus of G_d must intersect X. (Hint: If $V(G_d)$ is disjoint from X, use this to define a non-constant function on some connected component.)

4. Let X be a quasi-projective variety. Prove that X does not admit an infinite chain of strictly decreasing Zariski closed subsets.

5. For every positive integer n, let L_n be the line in $\mathbb{P}^3$ defined by

$$\left\{ [x, y, nx, ny] \;\middle|\; [x, y] \in \mathbb{P}^1 \right\}$$

Find the equations defining the Zariski closure of the infinite union

$$L_1 \cup L_2 \cup L_3 \cup \ldots$$

6. Let $X = \mathbb{P}^2 \setminus \{\text{point}\}$. Is there a non-constant algebraic map $f : X \to \mathbb{C}$?

Due April 15.