Exercise Sheet 11

1. a) Let S be a complex manifold. We want to consider families of geometric objects over S. Give a definition of a family of

   a) complex tori;
   b) abelian varieties;
   c) polarized abelian varieties.

b) Formulate an algebraic analogue: what is a family of abelian varieties over a complex variety S? This is called an *abelian scheme* over S.

c) Let X be a complex abelian variety. Show: there exists an irreducible complex variety S, an abelian scheme $\mathcal{X} \xrightarrow{p} S$ and two points $s_0, s_1 \in S$ such that $p^{-1}(s_0) = X$ and $p^{-1}(s_1)$ is a power of an elliptic curve with complex multiplication.