ETH Zürich | D-MATH | Differential Geometry II
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Exercise Sheet 10

Exercise 1

Let (M, g) be a Riemannian manifold with Riemannian curvature tensor R. In local coordinates,

$$R(\partial_i, \partial_j)\partial_k = \sum_{l=1}^{m} R_{ijk}\partial_l$$

Determine R^l_{ijk} in terms of the Christoffel symbols.

Exercise 2

Compute the Riemannian curvature thensor for the hyperbolic plane and determine the (constant) value of its sectional curvature.

Exercise 3

Let N and M be Riemannian manifolds and let $h : N \to M$ be a smooth map. Further, let $h^*(\mathrm{T}M)$ be the pullback of $\mathrm{T}M$ under h. Show the existence and uniqueness of a bilinear map

$$\nabla^h : \Gamma(\mathrm{T}N) \times \Gamma(h^*(\mathrm{T}M)) \to \Gamma(h^*\mathrm{T}M)$$

satisfying Proposition 3.12 of the lecture.

Exercise 4

Prove Proposition 3.13 of the lecture. Consider the following hint: Prove that $\nabla^h_X \overline{Y} - \nabla^h_Y \overline{X} - \overline{[X, Y]}$ is $C^\infty(N)$-linear in both variables and conclude by evaluating on coordinate vector fields.