

Exercise Sheet 6

Exercise 1

Show that the defining formula for the Levi-Civita connection developed in Theorem 2.30 of the lecture does indeed yield a connection.

Exercise 2

Let N be a smooth manifold and let $M \subseteq N$ be a regular submanifold of N . Show that every smooth vector field X on M extends to a smooth vector field on an open neighbourhood of M in N .

Exercise 3

Let M and N be smooth manifolds and let $f : M \rightarrow N$ be a smooth map. Show that

$$f^*TN := \{(p, v) \in M \times TN \mid v \in T_{f(p)}N\}$$

can be equipped with the structure of a smooth vector bundle with base M .