

## Exercise Sheet 9

### Exercise 1

Prove Corollary 2.63 of the lecture, giving a criterion for a piecewise  $C^1$ -curve to be a geodesic.

### Exercise 2

Show that  $\mathrm{SL}(2, \mathbb{R})$  does not admit a bi-invariant metric.

### Exercise 3

A Riemannian manifold  $M$  is said to be *homogeneous* if for any two points  $p, q \in M$  there is an isometry  $f$  with  $f(p) = q$ . Show that a homogeneous manifold is complete.