D-MATH Expander Graphs and Their Applications FS 16
Prof. Dr. Emmanuel Kowalski

Exercise Sheet 7

1. a) Show that the integers $\mathbb{Z}$ do not have property (T).

b) Let G and H be finitely generated (discrete) countable groups. Let $\varphi\colon G \longrightarrow H$ be a surjective homomorphism. Show that if G has property (T) then also H has property (T).

2. Let A be a finitely generated (discrete) abelian group. Show that A has property (T) if and only if $|A| < +\infty$.

3. Let G be a finitely generated (discrete) countable group. Assume that G has property (T). Let $H \triangleleft G$. Then G/H has also property (T).

4. Show that for any $k \geq 1$, the free group on k generators does not have property (T).

5. Let G be a finitely generated (discrete) countable group. Assume that G has property (T) and let $H < G$ be a finite index subgroup. Show that H has property (T).

6. Consider the subgroup

$$\Gamma_3 = \left\langle \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \right\rangle \subset \mathrm{SL}_2(\mathbb{Z})$$

Show that Γ_3 is a free group of rank 2.

7. Prove that $\mathrm{SL}_2(\mathbb{Z})$ does not have property (T).

Submission: Wednesday, 27th April 2016 during the exercise class.