

Graph Theory

Instructor: Benny Sudakov

Assignment 1

To be completed by March 3

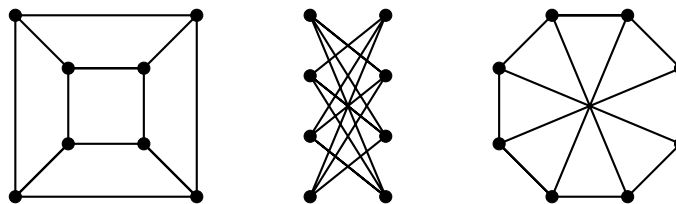
Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Given a graph G with vertex set $V = \{v_1, \dots, v_n\}$ we define the *degree sequence* of G to be the list $d(v_1), \dots, d(v_n)$ of degrees in decreasing order. For each of the following lists, give an example of a graph with such a degree sequence or prove that no such graph exists:

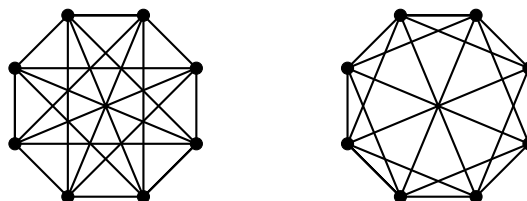
- (a) 3, 3, 2, 2, 2, 1
- (b) 6, 6, 6, 4, 4, 2, 2
- (c) 6, 6, 6, 6, 5, 4, 2, 1
- (d) 6, 6, 6, 4, 4, 3, 3

Problem 2:

- (a) Which of the following graphs are isomorphic? Why?



- (b) Are the following graphs isomorphic?



Problem 3: Prove that if a graph G is not connected then its complement \overline{G} is connected. Is the converse also true?

Problem 4: Show that every graph on at least two vertices contains two vertices of equal degree.

Problem 5: Prove that every graph with $n \geq 7$ vertices and at least $5n - 14$ edges contains a subgraph with minimum degree at least 6.

Problem 6: Show that in a connected graph any two paths of maximum length share at least one vertex.

Problem 7: Prove that a graph is bipartite iff (if and only if) it contains no cycle of odd length.