

Graph Theory

Instructor: Benny Sudakov

Assignment 10

To be completed by May 12

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Find the number of spanning trees of $K_n - e$ (the complete graph on n vertices with one edge removed) in two different ways:

- (a) using the Matrix Tree Theorem, and
- (b) using a double counting argument.

Problem 2: Is it true that if every vertex of a tournament has positive in- and out-degree, then the tournament contains a Hamilton cycle?

Problem 3: Suppose G is a graph on n vertices where all the degrees are at least $\frac{n+q}{2}$. Show that any set F of q independent edges is contained in a Hamiltonian cycle.

[Hint (step 1): .elpmaxeretnuoc lamixam(-egde) a ekat dna ,ton esoppuS]

[Hint (step 2): .egde siht diova ot noitator a esU .elcyc-H doog a setaerc egde na gniddA]