Graph Theory

Instructor: Benny Sudakov

Assignment 3

To be completed by March 17

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Prove that a connected graph G is k-edge-connected if and only if each block of G is k-edge-connected.

Problem 2: Let G be a graph and suppose some two vertices $u, v \in V(G)$ are separated by $X \subseteq V(G) \setminus \{u, v\}$. Show that X is a minimal separating set (i.e. there is no proper subset $Y \subsetneq X$ that separates u and v) if and only if every vertex in X has a neighbor in the component of $G - X$ containing u and another in the component containing v.

Problem 3: Show that if G is a graph with $|V(G)| = n \geq k + 1$ and $\delta(G) \geq (n + k - 2)/2$ then G is k-connected.

Problem 4: Prove that a graph G is 2-connected if and only if for any three vertices x, y, z there is a path from x to z containing y.

Problem 5: Let G be a k-connected graph, where $k \geq 2$. Show that if $|V(G)| \geq 2k$ then G contains a cycle of length at least $2k$.

[Hint: .ti dnetxe ot yrt dna htgnel mumixam fo elcyc a ekaT]