

Graph Theory

Instructor: Benny Sudakov

Assignment 7

To be completed by April 21

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Are the following statements true?

- (a) If G and H are graphs on the same vertex set, then $\text{dg}(G \cup H) \leq \text{dg}(G) + \text{dg}(H)$.
- (b) If G and H are graphs on the same vertex set, then $\chi(G \cup H) \leq \chi(G) + \chi(H)$.
- (c) Every graph G has a $\chi(G)$ -coloring where $\alpha(G)$ vertices get the same color.

Problem 2: G has the property that any two odd cycles in it intersect (they share at least one vertex in common). Prove that $\chi(G) \leq 5$.

[Hint: .hparg etitrapib a teg ot secitrev emos evomeR]

Problem 3: For a vertex v in a connected graph G , let G_r be the subgraph of G induced by the vertices at distance r from v . Show that $\chi(G) \leq \max_{0 \leq r \leq n} \chi(G_r) + \chi(G_{r+1})$.

Problem 4: Let l be the length of the longest path in a graph G . Prove $\chi(G) \leq l + 1$ using the fact that if a graph is not d -degenerate then it contains a subgraph of minimum degree at least $d + 1$.

Problem 5: Suppose the complement of G is bipartite. Show that $\chi(G) = \omega(G)$.

[Hint: tmemelpmoc etitrapib sti fo ytreporp a ot G fo ytreporp siht etalsnarT
.esruoc eht morf tluser reilrae na esu dna]