Graph Theory

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Assignment 8

To be completed by April 28

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: For a given natural number n, let G_n be the following graph with $\binom{n}{2}$ vertices and $\binom{n}{3}$ edges: the vertices are the pairs (x, y) of integers with $1 \leq x < y \leq n$, and for each triple (x, y, z) with $1 \leq x < y < z \leq n$, there is an edge joining vertex (x, y) to vertex (y, z). Show that for any natural number k, the graph G_n is triangle-free and has chromatic number $\chi(G_n) > k$ provided $n > 2^k$.

[Hint: .tcudni dna $\{k$ roloc sah (y, x) on $: x\}$ dna $\{k$ roloc sah (x, y) on $: x\}$ stes eht enifeD]

Problem 2: Show that the theorem of Mader implies the following weakening of Hadwiger's conjecture: Any graph G with $\chi(G) \geq 2^{t-2} + 1$ has a K_t-minor.

Problem 3: Find the edge-chromatic number of K_n (don't use Vizing's theorem).

[Hint: tsrif n ddo oD]

Problem 4: Let G be a connected k-regular bipartite graph with $k \geq 2$. Show, using Kőnig's theorem, that G is 2-connected.

Problem 5: Prove that every graph G of maximum degree Δ has an equitable $(\Delta + 1)$-edge-coloring, i.e. one where each color class contains $\lfloor e/(\Delta + 1) \rfloor$ or $\lceil e/(\Delta + 1) \rceil$ edges, where e is the number of edges in G.