

Graph Theory

Instructor: Benny Sudakov

Assignment 9

To be completed by May 6

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Show that $\chi(C_n) = \chi_l(C_n)$ for any $n \geq 3$.

Problem 2: Let G be a bipartite graph on n vertices. Prove that $\chi_l(G) \leq \lceil \log_2 n \rceil$ using the probabilistic method.

[Hint: xetrev hcae gniroloc yrt dna sessalc owt eht fo eno ot ylmodnar roloc hcae ngissA .seruliaf fo rebmun eht tnuoc ot noitatcepxe fo ytiraenil esU .ssalc sti morf sroloc htiw]

Problem 3: Let G be a complete r -partite graph with all parts of size 2. (In other words, G is K_{2r} minus a perfect matching.) Show, using a combination of induction and Hall's theorem, that $\chi_l(G) = r$.

[Hint: .stsil tniojsid evah secitrev tnecajda-non emussa yam ew taht wohs :dnuob reppU]

Problem 4: How many spanning trees does $K_{r,s}$ have?