Graph Theory

Solutions 11

The purpose of this write-up is merely to provide some guideline on how solutions should look like, and to help clean up hazy arguments. For hints for other problems, feel free to consult your teaching assistant.

Problem 2:

(a) sketch: Suppose G contains a K_4-subdivision. First observe that if this subdivision is not isomorphic to K_4 (i.e. at least one edge is subdivided) then it contains a $K_{2,3}$-subdivision. Otherwise there is a vertex v not contained in this copy of K_4, and two disjoint paths connecting v to the copy. Find the $K_{2,3}$-subdivision in this, as well, to get a contradiction.

(b) Add a vertex connected to all other vertices. The resulting graph will not have any $K_{3,3}$ or K_5 subdivision, hence it is planar. Thus it is a graph on $n + 1$ vertices having at most $3(n + 1) - 6 = 3n - 3$ edges. We added n edges to the original graph, so that one had at most $2n - 3$ edges.