Graph Theory

Solutions 12

The purpose of this write-up is merely to provide some guideline on how solutions should look like, and to help clean up hazy arguments. For hints for other problems, feel free to consult your teaching assistant.

Problem 2:

Take $(p-1)^2$ vertices and split them into $p-1$ equal groups. Now we color each edge induced by some of the groups red, and color edges between different groups blue. We claim that this edge-coloring of the graph contains no monochromatic clique of size p. Indeed, among any p vertices, we will have two in the same group (by the pigeonhole principle), so any p vertices induce a red edge. On the other hand, two of the vertices must come from different groups, so a blue edge is also induced. Hence this construction shows $R(p,p) > (p-1)^2$.

Problem 3:

Let $n \geq R_r(3)$ (where $R_r(3)$ is the r-colored Ramsey number $R_r(3,\ldots,3)$) and suppose we are given some coloring of all the subsets of $[n]$ using r colors. We define an edge-coloring of the complete graph with vertex set $[n]$ by giving the edge ij (with $i < j$) the color of the set $\{i,\ldots,j-1\}$. As $n \geq R_r(3)$, we know that this graph contains a monochromatic triangle on some three vertices i,j,k (with $i < j < k$). But this means that the sets $X = \{i,\ldots j-1\}, Y = \{j,\ldots k-1\}$ and $X \cup Y = \{i,\ldots k-1\}$ have the same color, exactly what we wanted. So any $n \geq R_r(3)$ satisfies the requirements of the problem.