Graph Theory

Solutions 13

The purpose of this write-up is merely to provide some guideline on how solutions should look like, and to help clean up hazy arguments. For hints for other problems, feel free to consult your teaching assistant.

Problem 3:

Let s_i be the length of the longest decreasing subsequence starting at a_i. Notice that if $s_i = s_j$ for some $i < j$ then $a_i < a_j$. Indeed, otherwise we could add a_i to the decreasing sequence of length s_j starting at a_j and get a sequence of length s_i+1 starting at a_i, contradicting our definition.

Now if there is no decreasing subsequence of length $l + 1$, then all the s_i are in $\{1, \ldots, l\}$. There are $kl + 1$ numbers, so some value $x \in$ is taken by at least $k + 1$ of the s_i. But by the above observation, these s_i will form an increasing sequence of length $k + 1$, which is what we wanted to show.

An alternative, more compact, way of presenting the above solution could be to show that the map $i \to (s_i, t_i)$ — where s_i, t_i are the lengths of the longest decreasing/increasing subsequences starting at a_i, respectively — is injective, hence not all the (s_i, t_i) can lie in $[l] \times [k]$.