Exercise sheet 5

Due date: 13:00, April 11.

Location: Next to HG G 52.1.

Exercise 5.1 Let ψ given by (V_0, ϑ) be a self-financing strategy in a multi-period market with discounted asset prices $(1, X) = S/S^0$.

Assume that $V_T(\psi) \geq -a$ P-a.s. for some $a \geq 0$.

(a) Show that if the market is arbitrage free, then ψ is a-admissible, i.e., $V_k(\psi) \geq -a$ P-a.s. for all $k = 0, \ldots, T$.

(b) Show that if X admits an ELMM Q and $V_0 \in L^1(Q)$, then $V_k(\psi) \geq -a$ P-a.s. for all $k = 0, \ldots, T$.

Exercise 5.2 Let M be a local martingale which is bounded from below by $-a$ for some $a \geq 0$ and is integrable at the initial time: $M_0 \in L^1(P)$. Show from the definitions that M is a supermartingale.

Exercise 5.3 Let X be any adapted, integrable process.

(a) Show that there exist a martingale $M = (M_k)_{k \in \mathbb{N}_0}$ and a predictable process $A = (A_k)_{k \in \mathbb{N}}$ with $A_0 := 0$ and

$$X_k = X_0 + M_k + A_k \quad P\text{-a.s.},$$

for $k \in \mathbb{N}_0$ and $M_0 = A_0 = 0$ P-a.s.

(b) Show that M and A are unique up to identification P-a.s.

(c) Show that X is a supermartingale if and only if A is decreasing, i.e., $A_{k+1} \leq A_k$ P-a.s. for all $k \in \mathbb{N}_0$. (So then we can write $X = X_0 + M - B$ with an increasing predictable process B null at 0.)