Exercise sheet 6

Due date: 13:00, April 18.

Location: Next to HG G 52.1.

Exercise 6.1 Consider an arbitrage-free market with probability measure P.

(a) Show that if $R \approx P$, the market is still arbitrage-free if P is replaced by R.

(b) Show that this is not necessarily true with only $R \ll P$.

Exercise 6.2 Consider an arbitrage-free market on $(\Omega, \mathcal{F}, (\mathcal{F}_k)_{k=0,\dots,T}, P)$, with discounted asset prices $(1, X) = S/S^0$. Let

$$\mathcal{G}^a = \{G_T(\vartheta) | \vartheta \text{ is } a\text{-admissible}\}.$$

Then

$$\mathcal{G}_{\mathrm{adm}} = \bigcup_{a \geq 0} \mathcal{G}^a.$$

(a) Show that $\mathcal{G}_a$ is closed in $L^0(\mathcal{F}_T)$ for all $a \geq 0$.

(b) Show that $\mathcal{G}_{\mathrm{adm}}$ is not necessarily closed in $L^0(\mathcal{F}_T)$.
Hint: Create a market with $T = 2$, where the price increment in the second step is determined by the outcome from the first step.

Exercise 6.3 Consider the space $(\Omega, \mathcal{F})$ with filtration $(\mathcal{F}_k)_{k \in N_0}$ and the two locally equivalent measures P and Q. Show that if $Z^{Q;P}$ is the density process of Q with respect to P, i.e.,

$$Z_k^{Q;P} = \frac{\mathrm{d}Q|_{\mathcal{F}_k}}{\mathrm{d}P|_{\mathcal{F}_k}}$$

for all $k \in \mathbb{N}_0$, where $Q|_{\mathcal{F}_k}$ denotes the restriction of Q to $\mathcal{F}_k$, then

$$Z^{P;Q} = \frac{1}{Z^{Q;P}},$$

i.e., $\frac{1}{Z^{Q;P}}$ is the density process of P with respect to Q.