Exercise sheet 7

Due date: 13:00, April 25.

Location: Next to HG G 52.1.

Exercise 7.1 Construct probability measures P and Q such that $Q \overset{\text{loc}}{\approx} P$, but $Q \not\approx P$.

Exercise 7.2 In Exercise 4.3, we have introduced a multi-period binomial market. In a similar fashion, we construct a trinomial market: Fix $r > -1$ and let $S_k^0 = (1+r)^k$. Now define $S_0^1 = 1$ and

$$S_k^1 = \prod_{i=1}^{k} R_i^1, \quad k = 1, \dots, T,$$

where the R_k^1 are i.i.d. and

$$P[R_k^1 = 1+u] = p^u, \quad P[R_k^1 = 1+m] = p^m, \quad P[R_k^1 = 1+d] = p^d,$$

all > 0, for $u > m > d$ and $u > r > d$. Note that the superscipts do not indicate powers.

Find the set of *all* equivalent martingale measures. You may provide an answer with, e.g., $T = 2$.

Hint: Use $\Omega = \{u, m, d\}^T$.

Exercise 7.3 In this exercise, we consider the probability space $(\Omega, \mathcal{F}, P)$ with $\mathcal{F}_0$ trivial and $\mathcal{F}_1 = \mathcal{F} = \sigma(S_1^1)$ and with assets given by

$$S_0^0 = 1, \qquad S_0^1 = 1,$$
$$S_1^0 = e^r, \qquad S_1^1 = e^Y,$$

where Y follows a standard normal distribution under P.

(a) Show that Q given by

$$\frac{dQ}{dP} = \exp\left(-\left(\frac{1}{2} - r\right) Y - \frac{\left(\frac{1}{2} - r\right)^2}{2} \right)$$

is an equivalent martingale measure.

(b) Show that S^1/S^0 does not have a unique EMM by constructing an unattainable payoff.