Exercise sheet 8

Due date: 13:00, May 2.

Location: Next to HG G 52.1.

Exercise 8.1

(a) Consider a market without arbitrage. Show that for every (European) contingent claim $H \in L^0_+(\Omega, \mathcal{F}_T, P)$, there exists an equivalent martingale measure Q such that $H \in L^1(\Omega, \mathcal{F}_T, Q)$.

(b) Construct an example for a family of uniformly bounded random variables where the pointwise supremum is not a random variable.

Exercise 8.2 Suppose that Y, $Z > 0$ and YZ are all martingales. Give suitable additional assumptions under which

$$Y 1_{\{\cdot \leq k\}} + \frac{Z_{\cdot} Y_{\cdot}}{Z_k} 1_{\{\cdot > k\}}$$

is also a martingale for every k.

Exercise 8.3 Consider the trinomial market introduced in Exercises 2.3 and 7.2. Let $H \in L^0_+(F_T)$ be some contingent claim and denote by $U = (U_k)_{k=0,\ldots,T}$ the process given by

$$U_k = \operatorname*{ess\,sup}_{Q \in \mathbb{P}} E_Q[H|\mathcal{F}_k].$$

Assume that $m = r$.

(a) Suppose $T = 1$. Find an optional decomposition of U.

(b) Argue how to extend the result to multiple time periods.

(c) Show that in general the decomposition is not unique.