Sheet 6

1. Prove the following claims.

a) $\mathcal{L}^n$, Λ_F are Radon measures on $\mathbb{R}^n$ and $\mathbb{R}$, respectively.

b) $\mathcal{H}^s$ is not a Radon measure for $s < n$, but it is a Radon measure for $s \geq n$.

c) If μ is a Radon measure, $A \subset \mathbb{R}^n$ μ-measurable, then $\mu|A$ with

$$(\mu|A)(B) := \mu(A \cap B), B \subset \mathbb{R}^n$$

is a Radon measure as well.

2. Let $f : \Omega \subset \mathbb{R}^n \to \mathbb{R}$. Show that the following are equivalent.

i) $f^{-1}(U)$ is μ-measurable for every open $U \subset \mathbb{R}$

ii) $f^{-1}(B)$ is μ-measurable for every Borel set $B \subset \mathbb{R}$.

iii) $f^{-1}(]-\infty, a[)$ is μ-measurable for every $a \in \mathbb{R}$.

3. Let (X, μ, Σ) be a measure space and $f, g : X \to \mathbb{R}$ two measurable functions on X. Show: The sets $\{x; f(x) = g(x)\}$ and $\{x; f(x) < g(x)\}$ are measurable.

4. A function $g \colon \mathbb{R} \to \mathbb{R}$ is called Borel measurable, if for every open set $U \subset \mathbb{R}$ the set $g^{-1}(U)$ is a Borel set. Let (X, μ, Σ) be a measure space, let $f \colon X \to \mathbb{R}, g \colon \mathbb{R} \to \mathbb{R}$ be functions with f μ-measurable and g Borel-measurable. Show that $g \circ f$ is μ-measurable.

5. Let μ be a Borel measure on $\mathbb{R}$ and $f \colon [a, b] \to \mathbb{R}$ continuous μ-almost everywhere (i.e. the set of points where f is not continuous is a μ-zero-set). Show that f is μ-measurable.

6. Let μ be a Borel measure on $\mathbb{R}$. Show that every monotone function $f \colon [a, b] \to \mathbb{R}$ is μ-measurable.