Lecturer: Prof. Dr. Mete SONER
Coordinator: Yilin WANG

Exercise Series 11

Q1. Suppose that $X_1, \cdots, X_n$ form a random sample from a Poisson distribution for which the mean is unknown. Determine the maximum-likelihood estimator of the standard deviation of the distribution.

Q2. Suppose that $X_1, \cdots, X_n$ form a random sample from a distribution for which the p.d.f. $f(x|\theta)$ is as follows:

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1} & \text{for } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Also, suppose that the value of θ is unknown ($\theta > 0$). Find the M.L.E. of θ.

Q3. In a lake we want to estimate the amount of a certain type of fish. For this we mark 5 fishes and we let them mix with the others, when they are well mixed we fish 11, and we realize that there are 3 marked and 8 non-marked. What is the maximum-likelihood estimator for the amount of fishes?.

Q4. A gas station estimates that it takes at least α minutes for a change of oil. The actual time varies from costumer to costumer. However, one can assume that this time will be well represented by an exponential random variable. The random variable X, therefore, possess the following density funciont

$$f(t) = e^{\alpha - t}\mathbf{1}_{\{t \geq \alpha\}},$$

i.e. $X = \alpha + Z$ where $Z \sim Exp(1)$. The following values were recorded from 10 clients randomly selected (the time is in minutes):

$$4.2, 3.1, 3.6, 4.5, 5.1, 7.6, 4.4, 3.5, 3.8, 4.3.$$

Estimate the parameter α using the estimator of maximum likelihood.

Q5. Suppose that $X_1, \cdots, X_n$ form a random sample from a normal distribution for which both the mean and the variance are unknown. Find the M.L.E. of the 0.95 quantile of the distribution, that is, of the point θ such that $\mathbb{P}(X < \theta) = 0.95$.

The 0.95 quantile of a standard normal distribution is $1.645 =: \theta_0$.