

Lecturer: Prof. Dr. Mete SONER

Coordinator: Yilin WANG



## Exercise Series 12

- Q1.** If a random variable  $X$  has the  $\chi^2$  distribution with  $m$  degrees of freedom, then the distribution of  $X^{1/2}$  is called a *chi ( $\chi$ ) distribution with  $m$  degrees of freedom*. Determine the mean of this distribution.
- Q2.** Moore and McCabe (1999) describe an experiment conducted in Australia to study the relationship between taste and the chemical composition of cheese. One chemical whose concentration can affect taste is lactic acid. Cheese manufacturers who want to establish a loyal customer purchases the cheese. The variation in concentrations of chemicals like lactic acid can lead to variation in the taste of cheese. Suppose that we model the concentration of lactic acid in several chunks of cheese as independent normal random variables with mean  $\mu$  and variance  $\sigma^2 = 0.09$ .

Suppose that we will sample 20 chunks of cheese. Let

$$T = \sum_{i=1}^{20} (X_i - \mu)^2 / 20,$$

where  $X_i$  is the concentration of lactic acid in  $i$ th chunk. What number  $c$  satisfies  $\mathbb{P}(T \leq c) = 0.9$ ?

- Q3.** When the motion of a microscopic particle in a liquid or a gas is observed, it is seen that the motion is irregular because the particle collides frequently with other particles. The probability model for this motion, which is called *Brownian motion*, is as follows: A coordinate system is chosen in the liquid or gas. Suppose that the particle is at the origin of this coordinate system at time  $t = 0$ , and let  $(X, Y, Z)$  denote the coordinates of the particle at any time  $t > 0$ . The random variables  $X, Y$ , and  $Z$  are i.i.d., and each of them has the normal distribution with mean 0 and variance  $\sigma^2 t$ . Find the probability that at time  $t = 2$  the particle will lie within a sphere whose center is at the origin and whose radius is  $4\sigma$ .
- Q4.** Take  $x \in [0, 1]$ . We say that  $x$  is normal if for  $x$  in its binary form,

$$x = \sum_{n \in \mathbb{N}} x_n 2^{-n} \quad x_n \in \{0, 1\},$$

we have that  $\lim_{n \rightarrow \infty} \frac{|\{1 \leq k \leq n : x_k = 1\}|}{n} = \frac{1}{2}$ .

- (a) Prove that if we have a sequence  $(U_n)_{n \in \mathbb{N}}$  i.i.d. Bernoulli with parameter  $\frac{1}{2}$ , then  $U = \sum_{n \in \mathbb{N}} U_n 2^{-n}$  is a uniform random variable in  $[0, 1]$
- (b) Prove that if  $U \sim U(0, 1)$ ,  $\mathbb{P}(U \text{ is normal}) = 1$ .