Lecturer: Prof. Dr. Mete SONER
Coordinator: Yilin WANG

ETH Zürich

Exercise Series 13

Q1. We toss 100 times a coin and we get 60 head. We want to do a test to know whether the coin is fair.

(a) Test the hypothesis with a 0.01 level of significance. Should this test be one or two-tailed?.

(b) What is the biggest amount of head should we have in 100 tossings so we cannot discard $H_0 :=$ "The coin biased towards tail".

(c) Calculate all p_0 so that the null hypothesis

$$H_0(p_0) := \text{"Probability of head is } p_0 \text{ "},$$

would not be rejected in a test with 0.05 level of significance.
Hint: It will be useful to use the central limit theorem in all of this question.

Q2. Let δ be a test procedure and S_1 the critical region of δ, Θ the parameter space. The function $\pi(\theta|\delta)$, called the *power function* of the test δ, is determined by the relation

$$\pi(\theta|\delta) = \mathbb{P}(X \in S_1|\theta), \quad \text{for } \theta \in \Theta.$$

Suppose that $X_1, \cdots, X_n$ form a random sample from the normal distribution with unknown mean μ and known variance 1, and it is desired to test the following hypotheses:

$$\begin{aligned} H_0 &: \quad 0.1 \leq \mu \leq 0.2 \\ H_1 &: \quad \mu < 0.1 \text{ or } \mu > 0.2. \end{aligned}$$

Consider a test procedure δ such that the hypothesis H_0 is rejected if either $\overline{X_n} \leq c_1$ or $\overline{X_n} \geq c_2$, and let $\pi(\mu|\delta)$ denote the power function of δ. Suppose that the sample size is $n = 25$. Determine the values of the constants c_1 and c_2 such that

$$\pi(0.1|\delta) = \pi(0.2|\delta) = 0.07.$$