If $\frac{\partial F}{\partial t} = f\nu := \lambda H^{\alpha}\nu$. Then

- $\frac{d}{dt} g_{ij} = 2f h_{ij}$ • $\frac{d}{dt} g^{ij} = -2f h^{ij}$

- $\frac{d}{dt}(d\mu) = f H \, d\mu$ • $\frac{d}{dt}\nu = -\nabla f$

- $$\frac{d}{dt} h_{ij} = -\lambda\alpha H^{\alpha-1}\Delta h_{ij} - \lambda\alpha(\alpha-1) H^{\alpha-2}\nabla_i H \nabla_j H + \lambda(\alpha+1) H^{\alpha} h_i^{\,a} h_{aj} - \lambda\alpha H^{\alpha-1}|A|^2 h_{ij}$$

- $$\frac{d}{dt} H = -\lambda\alpha H^{\alpha-1}\Delta H - \lambda\alpha(\alpha-1) H^{\alpha-2}|\nabla H|^2 - \lambda |A|^2 H^{\alpha}$$

- $$\frac{d}{dt} h^i_j = -\lambda\alpha H^{\alpha-1}\Delta h_{ij} - \lambda\alpha(\alpha-1) H^{\alpha-2}\nabla_i H \nabla_j H + \lambda(\alpha-1) H^{\alpha} h_i^{\,a} h_{aj} - \lambda\alpha H^{\alpha-1}|A|^2 h_{ij}$$

- $$\frac{d}{dt}|A|^2 = -\lambda\alpha H^{\alpha-1}\Delta|A|^2 + 2\lambda\alpha H^{\alpha-1}|A|^2(1-|A|^2) + 2\lambda(\alpha-1)H^{\alpha-2}\left(H^2 h^{ia}h_{aj} - \alpha\nabla^i H \nabla_j H\right)h_i^{\,j}$$

MCF: $f = -H \quad (\alpha = 1; \ \lambda = -1)$

$$\boxed{\frac{dH}{dt} = \Delta H + |A|^2 H}$$

$$\frac{d|A|^2}{dt} = \Delta |A|^2 + 2(|\nabla A|^2 - 1)|\nabla A|^2$$

IMCF: $f = H^{-1} \quad (\alpha = -1; \ \lambda = 1)$

$$\boxed{\frac{dH}{dt} = \frac{1}{H^2}\Delta H - \frac{2|\nabla H|^2}{H^3} - \frac{|A|^2}{H}}$$

$$\frac{d|A|^2}{dt} = \frac{1}{H^2}\Delta |A|^2 + 2(|\nabla A|^2 - 1)\frac{|\nabla A|^2}{H^2}$$

$$- 4H^{-1}\operatorname{tr}(A^3) - 4H^{-3}\nabla H A \nabla H$$

$$\frac{d}{dt} g_{ij} = \frac{d}{dt} \left\langle \frac{\partial F}{\partial x^i}, \frac{\partial F}{\partial x^j} \right\rangle$$

$$= \left\langle \frac{\partial}{\partial x^i}(f\nu), \frac{\partial F}{\partial x^j} \right\rangle + \left\langle \frac{\partial F}{\partial x^i}, \frac{\partial}{\partial x^j}(f\nu) \right\rangle$$

$$= f \left\langle \frac{\partial \nu}{\partial x^i}, \frac{\partial F}{\partial x^j} \right\rangle + f \left\langle \frac{\partial F}{\partial x^i}, \frac{\partial \nu}{\partial x^j} \right\rangle$$

$$= 2f h_{ij} \quad //$$

$$0 = \frac{d}{dt}\left(\delta^a{}_b\right) = \frac{d}{dt}\left(g^{ak} g_{kb}\right)$$

$$= \left(\frac{d}{dt} g^{ak}\right) g_{kb} + g^{ak}\left(2f h_{kb}\right)$$

$$\Rightarrow \frac{d}{dt} g^{ij} = \left(\frac{d}{dt} g^{ik}\right) g_{kb} g^{bj}$$

$$= -2f h^{ij} \quad //$$

$$\frac{d}{dt}(d\mu) = \frac{d}{dt}\sqrt{\det g_{ij}}\, dx$$

$$= \frac{1}{2}(\det g_{ij})^{-1/2} (\det g_{ij})\, tr\left(g^{-1}\frac{d}{dt}g\right) dx$$

$$= \frac{1}{2}\sqrt{\det g_{ij}}\; g^{kl}\frac{d}{dt}g_{kl}\; dx$$

$$= \frac{1}{2} g^{kl}(2f\, h_{kl})\, d\mu$$

$$= f H\, d\mu \quad //$$

$$\frac{d}{dt}\nu = g^{ij}\left\langle \frac{d}{dt}\nu, \frac{\partial F}{\partial x^i}\right\rangle \frac{\partial F}{\partial x^j}$$

$$= -g^{ij}\left\langle \nu, \frac{\partial}{\partial x^i}(f\nu)\right\rangle \frac{\partial F}{\partial x^j}$$

$$= -g^{ij}\frac{\partial f}{\partial x^i}\frac{\partial F}{\partial x^j}$$

$$= -\nabla f \quad //$$

$$\frac{d}{dt} h_{ij} = \frac{d}{dt}\left\langle -\frac{\partial^2 F}{\partial x^i \partial x^j}, \nu \right\rangle$$

$$= -\left\langle \frac{\partial^2}{\partial x^i \partial x^j}(f\nu), \nu\right\rangle - \left\langle \frac{\partial^2 F}{\partial x^i \partial x^j}, g^{ab}\frac{\partial f}{\partial x^a}\frac{\partial F}{\partial x^b}\right\rangle$$

$$= -\frac{\partial^2 f}{\partial x^i \partial x^j} - f\left\langle \frac{\partial}{\partial x^i}\left(\frac{\partial \nu}{\partial x^j}\right), \nu\right\rangle$$

$$+ \left\langle \Gamma_{ij}^{k}\frac{\partial F}{\partial x^k} - h_{ij}\nu, g^{ab}\frac{\partial f}{\partial x^a}\frac{\partial F}{\partial x^b}\right\rangle$$

$$= -\frac{\partial^2 f}{\partial x^i \partial x^j} - f\left\langle \frac{\partial}{\partial x^i}\left(h_{jr} g^{rs}\frac{\partial F}{\partial x^s}\right), \nu\right\rangle$$

$$+ \Gamma_{ij}^{k} g^{ab}\frac{\partial f}{\partial x^a} g_{kb}$$

$$= -\frac{\partial^2 f}{\partial x^i \partial x^j} - f h_{jr} g^{rs}\left\langle \Gamma_{is}^{l}\frac{\partial F}{\partial x^l} - h_{is}\nu, \nu\right\rangle$$

$$+ \Gamma_{ij}^{a}\frac{\partial f}{\partial x^a}$$

$$= -\nabla_i \nabla_j f + f h_{is} h_j^{\,s}$$

$$\Delta h_{ij} = g^{kl} \nabla_k \nabla_l h_{ij}$$

$$\text{Codazzi} \curvearrowright = g^{kl} \nabla_k \nabla_i h_{lj}$$

$$= g^{kl} \left[\nabla_i \nabla_k h_{lj} + Rm_{ki\ \,l}^{\ \ \,a} h_{aj} + Rm_{ki\ \,j}^{\ \ \,a} h_{la} \right]$$

$$\begin{matrix}\text{Gauss} \\ + \\ \text{Codazzi}\end{matrix} \curvearrowright = g^{kl} \nabla_i \nabla_j h_{kl} + g^{kl} \left(h_{kl} h_i^{\ a} - h_k^{\ a} h_{il} \right) h_{aj}$$

$$+ g^{kl} \left(h_{kj} h_i^{\ a} - h_k^{\ a} h_{ij} \right) h_{la}$$

$$= \nabla_i \nabla_j H + H h_i^{\ a} h_{aj} - |A|^2 h_{ij}$$

$$f := \lambda H^{\alpha} \curvearrowright = \frac{\nabla_i \nabla_j f}{\lambda \alpha H^{\alpha - 1}} - \frac{\alpha - 1}{H} \nabla_i H \nabla_j H + H h_i^{\ a} h_{aj} - |A|^2 h_{ij}$$

$$\Longrightarrow \frac{d}{dt} h_{ij} = -\nabla_i \nabla_j f + f\, h_{is} h_j^{\ s}$$

$$= -\lambda\alpha H^{\alpha-1}\Big(\Delta h_{ij} + \frac{\alpha-1}{H} \nabla_i H \nabla_j H$$

$$- H h_i^{\ a} h_{aj} + |A|^2 h_{ij} \Big)$$

$$+ \lambda H^{\alpha} h_{is} h_j^{\ s}$$

$$= -\lambda\alpha H^{\alpha-1} \Delta h_{ij} - \lambda\alpha(\alpha-1) H^{\alpha-2} \nabla_i H \nabla_j H$$

$$+ \lambda H^{\alpha} (\alpha+1) h_i^{\ a} h_{aj} - \lambda\alpha H^{\alpha-1} |A|^2 h_{ij}$$

//

$$\Rightarrow \frac{d}{dt} H = \left(\frac{d}{dt} g^{ij} \right) h_{ij} + g^{ij} \left(\frac{d}{dt} h_{ij} \right)$$

$$= \left(-2 f h^{ij} \right) h_{ij}$$

$$+ g^{ij} \Big(-\lambda \alpha H^{\alpha-1} \Delta h_{ij} - \lambda \alpha (\alpha-1) H^{\alpha-2} \nabla_i H \nabla_j H$$

$$+ \lambda H^{\alpha} (\alpha+1) h_{ia} h^a_j$$

$$- \lambda \alpha H^{\alpha-1} |A|^2 h_{ij} \Big)$$

$$= -2\lambda H^{\alpha} |A|^2 - \lambda \alpha H^{\alpha-1} \Delta H$$

$$- \lambda \alpha (\alpha-1) H^{\alpha-2} |\nabla H|^2 + \lambda (\alpha+1) H^{\alpha} |A|^2$$

$$- \lambda \alpha H^{\alpha-1} |A|^2 \cdot H$$

$$= -\lambda \alpha H^{\alpha-1} \Delta H - \lambda \alpha (\alpha-1) H^{\alpha-2} |\nabla H|^2 - \lambda |A|^2 H$$

$$\Longrightarrow \frac{d}{dt} h^i_{\ j} = \left(\frac{d}{dt} g^{ik}\right) h_{kj} + g^{ik} \left(\frac{d}{dt} h_{kj}\right)$$

$$= \left(-2 f h^{ik}\right) h_{kj}$$

$$+ g^{ik} \Big(-\lambda\alpha H^{\alpha-1} \Delta h_{kj} - \lambda\alpha(\alpha-1) H^{\alpha-2} \nabla_k H \nabla_j H$$

$$+ \lambda H^{\alpha} (\alpha+1) h_k^{\ a} h_{aj} - \lambda\alpha H^{\alpha-1} |A|^2 h_{kj} \Big)$$

$$= -\lambda\alpha H^{\alpha-1} \Delta h^i_{\ j} - \lambda\alpha(\alpha-1) H^{\alpha-2} \nabla^i H \nabla_j H$$

$$+ \lambda H^{\alpha} (\alpha-1) h^{ia} h_{aj} - \lambda\alpha H^{\alpha-1} |A|^2 h^i_{\ j}$$

$$\frac{d}{dt}|A|^2 = \left(\frac{d}{dt}h^i_j\right)h_i{}^j + h^i_j\left(\frac{d}{dt}h_i{}^j\right)$$

$$= -\lambda\alpha H^{\alpha-1}\left((\Delta h^i_j)h_i{}^j + h^i_j(\Delta h_i{}^j)\right)$$

$$-\lambda\alpha(\alpha-1)H^{\alpha-2}\left((\nabla^i H\nabla_j H)h_i{}^j + h^i_j\nabla_i H\nabla^j H\right)$$

$$+\lambda H^\alpha(\alpha-1)\left(h^{ia}h_{aj}h_i{}^j + h^i_j h_{ia}h^{aj}\right)$$

$$-\lambda\alpha H^{\alpha-1}|A|^2\left(h^i_j h_i{}^j + h^i_j h_i{}^j\right)$$

$$= -\lambda\alpha H^{\alpha-1}\Delta|A|^2 + 2\lambda\alpha H^{\alpha-1}\left(|\nabla A|^2 - |\nabla A|^4\right)$$

$$+2\lambda(\alpha-1)H^{\alpha-2}\left(H^2h^{ia}h_{aj} - \alpha\nabla^i H\nabla_j H\right)h_i{}^j$$