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Problem set 1

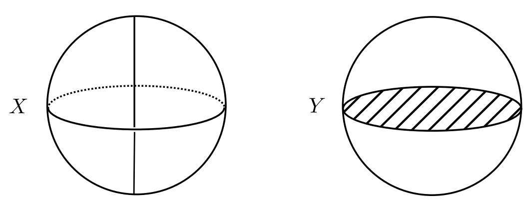
1. Suppose we are given maps $X' \xrightarrow{\phi} X \xrightarrow{f} Y$, where ϕ is a homotopy equivalence. Show that the induced map $F : C_{f\circ\phi} \to C_f$ is also a homotopy equivalence.

2. Let X be a contractible space and let $r : X \to A$ be a retraction. Show that A also is contractible.

3. Let $f, g : X \to S^n$ be maps such that $f(x) \neq -g(x)$ for all $x \in X$. Show that $f \simeq g$.

4. Let $f : X \to Y$ be a map. Suppose that there exist maps $g, h : Y \to X$ such that $f \circ g \simeq \mathrm{id}_Y$ and $h \circ f \simeq \mathrm{id}_X$. Show that f is a homotopy equivalence.

5. Consider the spaces X and Y depicted below. Show that $X \simeq S^2 \vee S^1$ and $Y \simeq S^2 \vee S^2$.



(In words: X is a 2-sphere with a stick connecting north and south pole; Y is a 2-sphere with a disc glued in along the equator.)

6. Consider the map $f : S^1 \to S^1$, $e^{2\pi it} \mapsto e^{4\pi it}$ (or $z \mapsto z^2$). Prove that $C_f \approx \mathbb{R}P^2$.