Commutative Algebra

D-MATH, HS 2013

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Exercise Sheet 6

Exercises **1. - 6.** are taken from the book *Introduction to Commutative Algebra* by Atiyah and Macdonald.

1. Let M_i ($i \in I$) be a family of A-modules and let M be their direct sum. Prove that M is flat if and only if M_i is flat for each i.

2. Let $A[x]$ be the ring of polynomials in one indeterminate over a ring A. Prove that $A[x]$ is a flat A-algebra.

3. Prove:

i) If M and N are flat A-modules for a ring A, then so is $M \otimes_A N$.

ii) If B is a flat A-algebra and N is a flat B-module, then N is flat as an A-module.

4. Let $A \subset B$ be two rings such that B is integral over A. Let $f : A \longrightarrow \Omega$ be a homomorphism of A into an algebraically closed field Ω. Show that f can be extended to a homomorphism of B into Ω. (Hint: Theorem 5.10 in the book)

5. Let $f : A \longrightarrow B$ be an integral homomorphisms of rings. Show that the induced map $f^* : \mathrm{Spec}(B) \longrightarrow \mathrm{Spec}(A)$ is a *closed* map, that is the image of a closed set is closed. (Hint: Use Theorem 5.10 from the book)

6. Let $A \subseteq B$ be rings with B integral over A. Show:

(i) If $x \in A$ is a unit in B, then it is a unit in A.

(ii) The Jacobson radical of A is the contraction of the Jacobson radical of B.

Due on Tuesday, 07.11.2013